

Mathematical Coding as an Application of Graph Theory: A Sustainable Approach to Urban Structure Analysis

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Abstract

This study investigates the application of graph theory through mathematical coding in structuring and developing pedestrian pathways within the complex morphology of traditional Arabic souqs (historic marketplaces). By employing Eulerian paths, space syntax analysis, and circulation assessments, we demonstrate how urban walkability can be enhanced through systematic network transformations. The research conceptualizes path labeling as a coding sequence, demonstrating how mathematical structures can reconstruct pedestrian networks to minimize discontinuity and maximize accessibility. Focusing on the historic Arabic souq in Baghdad, this study examines an urban pedestrian network shaped by organic alleyways, which have been disrupted by modern vehicular interventions. We assess how traditional pedestrian routes have been reconfigured, resulting in a reduction of their spatial integration. Through the graph theory-based transformation, we resolve odd-degree nodes, ensuring that pedestrian paths form Eulerian circuits with continuous, uninterrupted movement sequences. Applying space syntax, we measure the changes in spatial configuration revealing a recalibrated urban fabric with improved mobility and socio-spatial cohesion. Findings showed that leveraging graph-theoretic principles in urban design fosters sustainable pedestrian environments, reinstating historic circulation patterns and enhancing integration values. The results demonstrate a tangible improvement in spatial intelligibility and accessibility by ensuring seamless pedestrian flow. Unlike conventional vehicular centric planning, which often disrupts the historical spatial syntax, this approach offers an algorithmic strategy that balances heritage conservation with contemporary urban demands. By fostering urban walkability as a computational problem, this study underscores the transformative role of mathematical coding in urban planning. The research encourages the integration of graph theory into spatial design, proposing a novel methodology for reconciling modern mobility needs with the intrinsic logic of historic pedestrian networks.

Keywords

Graph theory; Eulerian paths; Space syntax; Urban walkability; Pedestrian networks; Sustainable urbanism

1. Introduction

The promotion of walkable urban environments has become a central objective in contemporary city planning, contributing to sustainability, social cohesion, and economic vitality (Caprotti et al., 2017). This is particularly critical in historic marketplaces (known as souqs in the Middle East and North Africa), which have long served as the commercial and social nuclei of cities (Khalid & Abaas, 2021). However, the modern transformation of urban networks, with a prioritization of vehicular infrastructure, has led to spatial fragmentation, congestion, and reduced pedestrian accessibility (Major & Tannous, 2020). This shift has significantly affected historic cores, where the reconfiguration of traditional pathways often disrupts the intelligibility and permeability of pedestrian movement (Eboli et al., 2023). Urban circulation is shaped by physical, behavioural, and environmental factors including street morphology, transportation systems, and environmental constraints. Natural features such as rivers, exemplified by the Tigris River in Baghdad, act as structural organizers of urban movement (Al Ani, 2014), while street network design and traffic behaviour further complicate pedestrian navigation (Alkaiissi, 2022). In response, sustainable urban development strategies seek to reconcile historic preservation with contemporary mobility needs, promoting walkability, accessibility, and resilience (Al-Saffar & Al-Siliq, 2014). Despite these efforts, systematic, computationally driven approaches remain underutilized in pedestrian network planning. This study investigates the application of graph theory and mathematical coding to optimize pedestrian circulation within traditional Arabic souqs. By identifying Eulerian paths and employing space syntax analysis, we assess how these methodologies can enhance connectivity, minimize disruption, and maintain the historic integrity of pedestrian environments.

This research is structured as follows: Section 2 introduces the case study, Section 3 shows the materials and methods, Section 4 outlines the main results of the analysis, and Section 5 presents conclusions. The study addresses two core questions: How does modifying pedestrian networks affect urban spatial configurations? And how can a continuous pedestrian route be designed to optimize accessibility without undermining traditional urban patterns?

2. Case Study: The Arabic Souq

The market or souq in Iraq historically played not only a local role but expanded to the regional scale. Moreover, historians have emphasized the strategic importance of Baghdad's location (the capital of Iraq) as a hub connecting east and west where convoys included the following regions: India, Iraq, Persia, and the Mediterranean Sea (Al Siliq, 2008). The traditional Arabic souq is located in the old Rusaffa side of eastern Baghdad (Albazzaz & Hussien, 2017). Old Rusaffa was founded during the caliphate of Al-Mahdi in the Abbasid era and became a distinctive part of Baghdad, serving as a centre of attraction surrounded by the Tigris River (Yousif & Abdulwahab, 2024). Old Rusaffa, the historical heart of Baghdad, features a distinctive architectural and cultural significance characterized by the traditional Arabic souq, Rashid Street, churches, courtyard houses, and significant mosques. Safeguarding these cultural and historical heritage sites will strengthen Baghdad's historical significance and preserve the city's identity (Echarri & Brebbia, 2018).

The study area is an urban fabric bounded by Ahrar Bridge to the east, Shouhadaa Street to the west, Jimhureya Street to the north, and the Tigris River to the south, as shown in Figure 1. The Arabic souq stimulates economic activity within the historic urban core (Alsaliq & Alshami, 2015). It typically consists of a maze of narrow, arcade-lined streets with housing, shops, and stalls, which conceal expansive warehouses and artists' workshops (Kirtikar, 2011). In Old Rusafa, the traditional souq emerged from a linear network of

shopping corridors, subdivided into distinct interconnected segments. Each section was secured by gates that were closed nightly, mirroring the controlled access seen in residential quarters. This modular design evolved further as replicated souq units were arranged “back-to-back” or expanded through perpendicular connections, forming intricate commercial complexes in the city centre (Al-Saffar, 2020). The souq’s layout aligned with primary thoroughfares which linked the city gates to its central hub, intersected by parallel alleys and cross-connections. These parallel pathways were organized into specialized zones that enabled the visitors to explore diverse goods within a unified space. The movement of people and merchandise between sections underscored the efficiency of this commercial network, a defining feature of traditional Muslim urban landscapes that shaped daily street activity (Bianca, 2000).

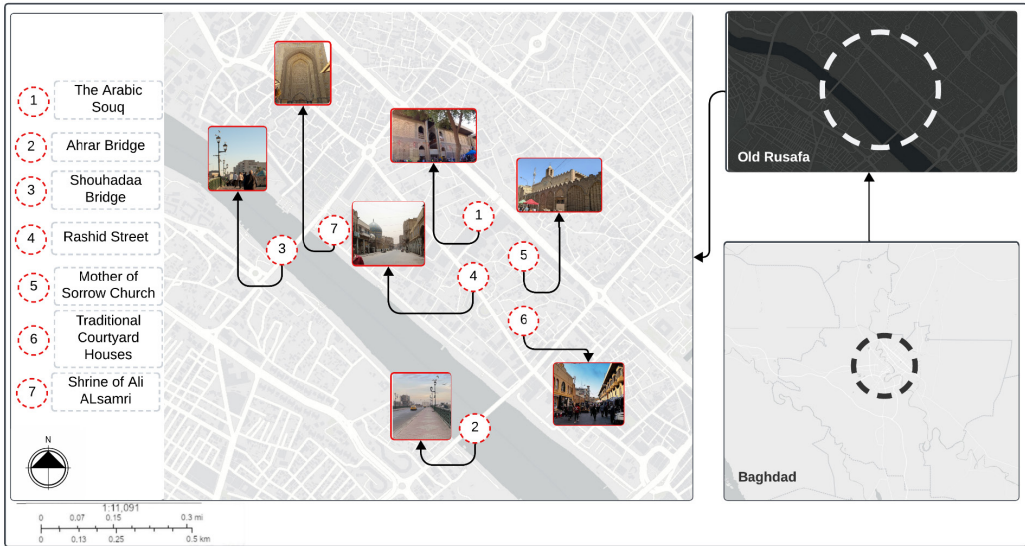


Figure 1. The traditional Arabic Souq area is located in the central business district area of Baghdad city (source: the authors using Esri ArcGIS maps).

3. Materials and Methods

Mathematical coding using graph theory was applied to the traditional Arabic souq to analyse the impact of graph theory on the development of pedestrian paths within the study area which combines pedestrian and automobile circulation as shown in Figure 2. We employed a mixed-method approach: 1) the determination of Eulerian paths, 2) the analysis of space syntax; and 3) an in-field assessment of circulation modes.

This analysis method was chosen to save cost and time while offering a comparison opportunity and Figure 3 summarizes the workflow for the data analysis and the methods used. First, identifying Eulerian paths within the study area establishes routes that traverse each segment exactly once, which eliminates redundancy. This approach optimizes pedestrian circulation within the alleyways by preventing overlapping paths and ensures efficient navigation.

Second, space syntax analysis evaluates the proposed algorithmic-based modifications derived from the mathematical coding, enabling a systematic comparison between the redesigned layout and the existing spatial configuration. Moreover, this step quantifies how structural adjustments influence connectivity and accessibility. Finally, in-field observational assessments measure the interplay between vehicular congestion, particularly during peak traffic periods, and pedestrian mobility which highlights the temporal constraints that affect walkability in the urban fabric

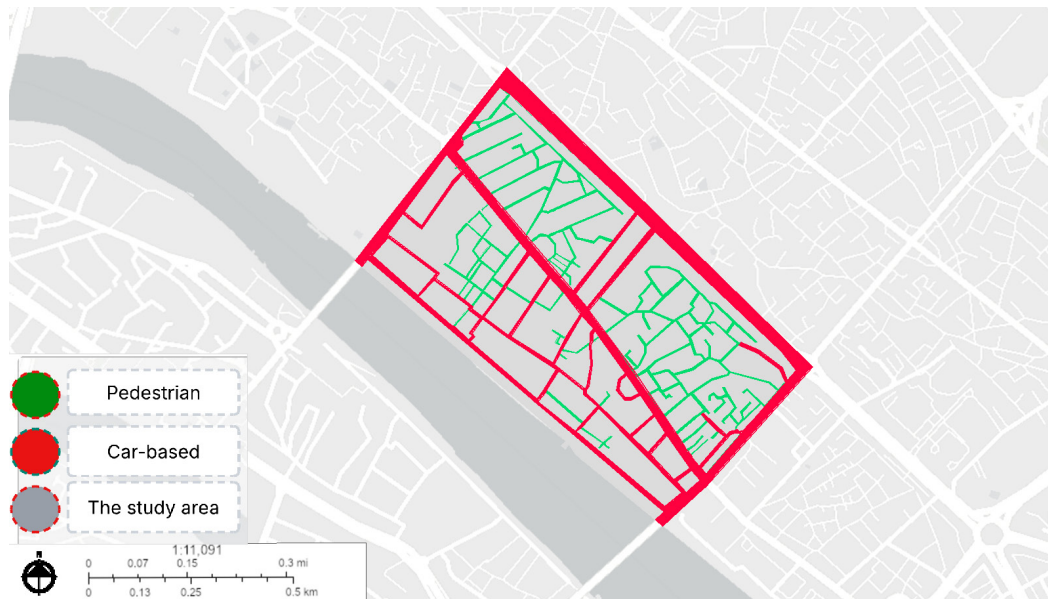


Figure 2. The pedestrian and car-based paths of the traditional Arabic souq area which indicate an intersection of both circulation modes. (source: the authors)

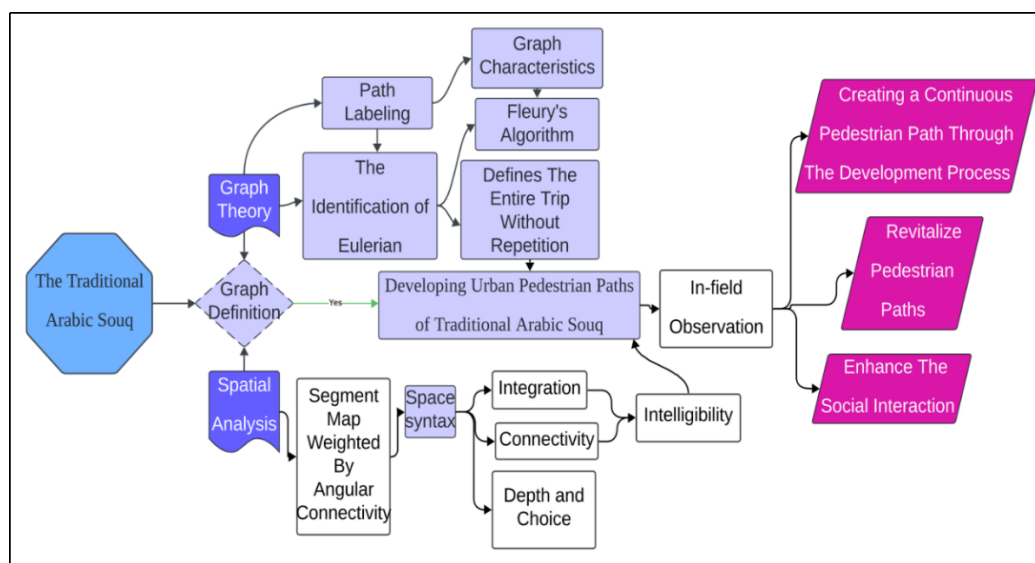


Figure 3. Workflow of methods and analysis. (source: the authors)

3.1 Previous Literature

A literature survey was conducted to articulate the concepts of coding, graph theory, and space syntax. The role of coding in various disciplines is described as a mathematical relationship (Table 1).

Table 1. Previous literature on coding concepts with relationship to scientific fields.

Coding and scientific fields	Coding and scientific fields	Coding and scientific fields Mathematical expression	Coding and scientific fields note
Coding and Cryptography	Method for communicating in secret	Morse code	(Yang et al., 2023)
Coding and Linguistics	Frequency of word use is inversely proportional to its rank	Zipfs law	(Bolea et al., 2024)
Coding and Biology	Markov chains express random phenomena evolving in time and use simple structures for various applications	Markov chain	(Carbonaro & Menale, 2024)
	Scaling law links all system component parts with a relational ratio, especially body mass. Scaling law is evident in dynamics and is characterized by self-similarity and complexity	Scaling law	(Ain et al.,2024; Han et al., 2024; Meckin, 2024)

3.1.1 Coding

Coding is an interdisciplinary concept; in language it is defined as a set of words, symbols, or phrases used to represent a secret form of messages. Codes consist of instructions that follow specific logical structures and can be generalized across disciplines (McIntosh et al., 2014). In computing, to code something is to create a computer software program by gathering one system of numbers, phrases, and signs into another system (I. Griffiths, 2024). The focus here is on coding the city (the built and natural environment, and the social content) into a software platform in order to manage the city and equip it with networks and technological devices (Kitchin & Perng, 2016). Biology has the term (genetic code) which refers to a person’s behaviours that are controlled genetically and coded into humans (Khan, 2024). Genetic coding in urban structure is found to achieve the resilient city using generative modules. Moreover, the purpose of coding preservation is to enhance the connection of the city (Qais & Al Ani, 2022). The following section reviews mathematical coding which is the subject of interest in this study.

3.1.2 Mathematical coding

Mathematical coding engages different theories to achieve several purposes, the game theory, for instance, is a set of mathematical formulas that describe economic behaviour (Barron, 2024). Game theory initially was developed to understand strategic interactions in games such as chess [Barron, 2024]. The puzzle of the bridge network on the Pregel River in the Prussian capital city was mathematically solved by Euler. The puzzle involved planning a walk that crosses all bridges exactly once (Königsberg bridge problem).

The starting point was the problem of graph theory which provides numerous measurements of connectivity patterns (Rozhkov, 2024). Abstracting the city and eliminating its features except the landmasses and the linking bridges, when dots, vertices or nodes replace the landmasses and edges or links replace the bridges. These are called topological properties of graphs (or networks) that are used to limit or enhance commuting and traveling within the city (Blanchard & Volchenkov, 2008). Graph theory is a practical branch of mathematics that considers the relationships of nodes and how they are arranged.

Graphs, representing relationships between objects, can be classified as directed or undirected (O'Regan, 2020). The topological qualities are non-geographic methods that focus on: spatial configuration, permeability, and complexity (Ostwald & Dawes, 2018). Game theory, chess, puzzles, and billiards are interpreted and used for prediction mathematically. Moreover, billiards are mathematical models for several physical phenomena that were introduced as mathematical chaotic theory in 1970. Chaotic billiards demand the understanding of linear algebra, topology, measure theory, complex analysis, and probability theory (Chernov & Markarian, 2023). Puzzle of Bridges also uses topology as a branch of mathematics that abstracts city elements to nodes and edges or graph theory (Nazar et al., 2024). Graph theory refers to the interpretation of graphs consisting of mathematical structures (Reba & Shier, 2014). Graph theory enables commuters to travel from one part of the city to the other (Biggs et al., 1986). Euler states that there is a possibility to cross all the bridges once and once only (Sandifer, 2007).

3.1.3 Space syntax

Space syntax theory enrolls a set of methods that provide a means of developing mathematical measures of the underlying spatial structures of the city rather than its Euclidean dimensions (Hiller & Hanson, 1984). This method abstracts the plans into a mathematical graph which can be analysed to articulate different characteristics of the underlying spatial structure of the city (Dawes & Ostwald, 2020; van Nes & Yamu, 2021). Space syntax embraces three techniques: convex space analysis, axial-segment line analysis, and visibility graph analysis (Dawes & Ostwald, 2013). The isovist analysis was elaborated by Michael Benedikt (Benedikt, 1979). The axial map analysis was pioneered by Bill Hillier (Hiller & Hanson, 1984). The convex map was explored by John Peponis (Peponis & Wineman, 2002). The justified plan graphs are used in graph theory, which is the study of mathematical structures through building relationships between objects where spaces are configured as nodes and the connections are edges (van Nes & Yamu, 2021). Space syntax as graph theory has several applications in urban design such as the urban public space (Freire de Almeida et al., 2021), urban artificial neural networks (Yönder et al., 2023), theoretical foundations (Hillier et al., 1976), historical urban research and graph spectra (S. Griffiths, 2012), environmental cognition (Penn, 2003), and topological networks (Dursun, 2007). Graph theory is related to space syntax through the spatial configuration of the urban structure that is governed by topological (non-geometric) relationships.

3.1.4 Network analysis in transportation planning

Graph theory is essential in transportation planning and analysis, providing a structured model approach for evaluating transportation systems (Said et al., 2022), including the optimization of routes, reduction in travel time, minimization of congestion, and upgrading the overall efficiency of transportation networks (Almasan et al., 2022). Network analysis is based on graph configurations where vertices represent urban intersections, terminals, transportation hubs, etc. (Boeing, 2022). Edges link pairwise objects (the nodes in this case) to illustrate roads, railways, or flight paths. Urban edges are characterized by distance, capacity, or travel time (Alessandretti et al., 2023). Transportation planning algorithms are extracted from graph theory such as shortest path algorithms, network flow analysis, centrality measures, network reliability and resilience, traffic flow models, public transit network design, and connectivity analysis (Amaya & Holweg, 2024; Gajjar et al., 2024).

3.2 Identification of Eulerian

We converted the study area map to a graph consisting of nodes and paths and employed prefix labelling as shown in Figure 4. The aim is to find the graph theory’s Eulerian in the map of the traditional Arabic souq area which combines both pedestrian and automobile circulation as shown in Figure 5. Converting the map to a graph means that pairs of vertices are joined by edges. Moreover, all edges connect the graph vertices. These kinds of graphs have applications in designing transportation systems to calculate distances or costs between the starting location and the destination using the graph (Taylor, 2004). The topology of a graph is the possibility of considering the plane diagram of it as a geographical map i.e. the polygon faces are the countries and the edges are the frontiers (Biggs et al., 1986). Euler’s path in a certain graph is a route which crosses each edge once and only once. Euler’s trail is a walk with no repeated edges (and visits every edge once), Euler’s tour is considered a closed Euler’s trail and if a graph has a Euler’s tour it is called Eulerian (Bahmanian & Shan, 2024). A connected graph W contains a Eulerian if and only if every vertex of the graph W has an even degree (Sivanesan et al., 2024). A connected graph W has an Euler’s trail if and only if it has at most two vertices of odd degrees (Hammack & Kainen, 2024).

Theorem: a finite graph W contains Euler’s path if and only if (Silberger, 2024):

- (1) W is connected: choosing the vertices for example (v and u) it is possible to find a path starting at v and ending at u . In contrast, a disconnected graph splits into connected parts or what is known as components.
- (2) The degree: the degree of a vertex (u) is the number of edges which connect at (u). Eulerian for example contains all local degrees in even numbers.
- (3) If the graph W has odd degree vertices (more than two), then it cannot contain an Eulerian.
- (4) If a connected graph W has no vertices of odd degree, then it contains an Eulerian.

Addressing the graph as Eulerian will need to turn odd-degree nodes to even degrees which will eliminate and add segments or paths. The next step is to insert the nodes with odd degrees into a table, as shown in Table 2. Since Eulerian is achieved by a graph with even degrees, we shall edit the graph by converting odd nodes to even by removing and adding segments as shown in Figure 6.

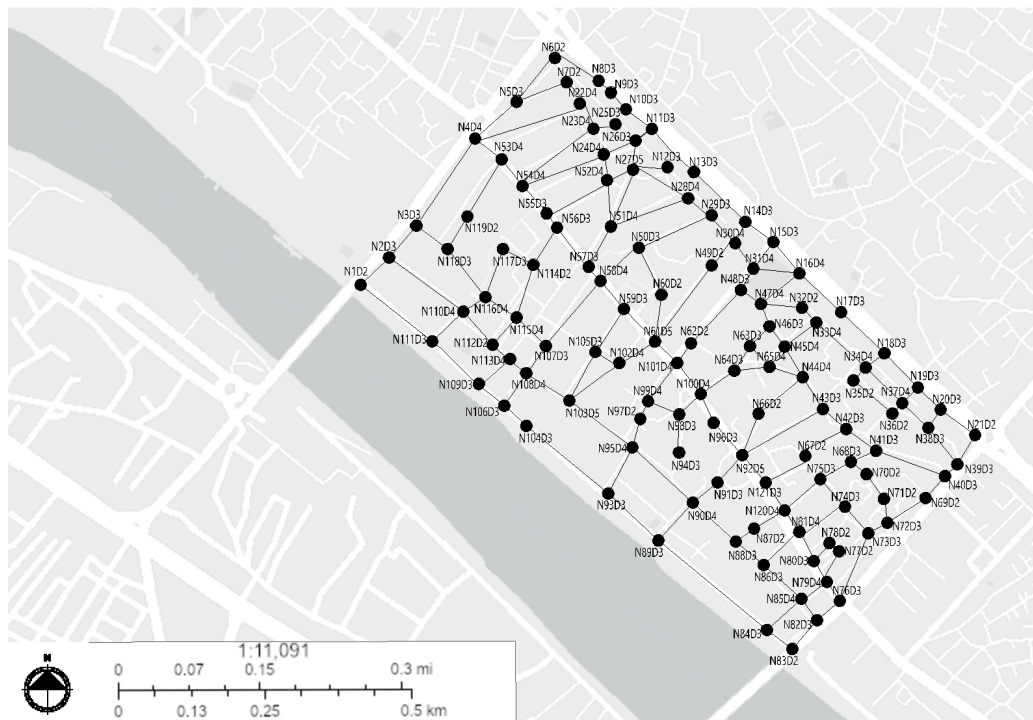


Figure 4. The prefix-labelling of nodes where N is for node and D is for the degree of the relevant node within the study area (source: the authors)



Figure 5. The traditional Arabic souq area combines both pedestrian and automobile circulation that influence the population's health and well-being. (source: the authors)

Table 2. Node N number and its corresponding degree D.

N1D2	N2D3	N3D3	N4D4	N5D3
N6D2	N7D2	N8D3	N9D3	N10D3
N11D3	N12D3	N13D3	N14D3	N15D3
N16D4	N17D3	N18D3	N19D3	N20D3
N21D2	N22D4	N23D4	N24D4	N25D3
N26D3	N27D5	N28D4	N29D3	N30D4
N31D4	N32D2	N33D4	N34D4	N35D2
N36D2	N37D4	N38D3	N39D3	N40D3
N41D3	N42D3	N43D3	N44D4	N45D4
N46D3	N47D4	N48D3	N49D2	N50D3
N51D4	N52D4	N53D4	N54D4	N55D3
N56D3	N57D3	N58D4	N59D3	N60D2
N61D5	N62D2	N63D3	N64D3	N65D4
N66D2	N67D2	N68D3	N69D2	N70D2
N71D2	N72D3	N73D3	N74D3	N75D3
N76D3	N77D2	N78D2	N79D4	N80D3
N81D4	N82D3	N83D2	N84D3	N85D4
N86D3	N87D27	N88D3	N89D3	N90D4
N91D3	N92D5	N93D3	N94D3	N95D4
N96D3	N97D2	N98D3	N99D4	N100D4
N101D4	N102D4	N103D5	N104D3	N105D3
N106D3	N107D3	N108D4	N109D3	N110D4
N111D3	N112D2	N113D4	N114D2	N115D4
N116D4	N117D3	N118D3	N119D2	N120D4
N121D3				

Fleury's algorithm was used to find Euler's tour or circuit; it is the algorithm of the nearest neighbour in the graph G. The circuit was formed by continuously assigning a rank to the closest neighbouring vertex (Jusriadi et al., 2024). We applied Fleury's algorithm to the graph and defined Euler's tour starting and ending with the node labelled (N103D5). The node (N103D5) is well-connected within the graph which ensures a feasible traversal of the entire network and it is characterized with a strategic place that minimizes the backtracking.

3.3 Space Syntax Analysis

Space syntax was used to analyse the study area before and after applying the graph theory (Eulerian) to examine its effect on pedestrian development. Space syntax is similar to graph theory that defines several relationships between streets in the spatial system (Hidayati et al., 2021). Intelligibility means the clarity of space or how it is more understandable (Sari et al., 2024). The segment analysis of the spatial network to calculate intelligibility contains connectivity and integration (Asriana et al., 2024).

3.4 In-field Assessment

Obtaining pedestrian mobility routes achieves the improvement and development of the area by proposing a pedestrian model (Altunina & Boikova, 2023). Redesigning streets to meet pedestrian needs will provide well-being, equity, liveability, and enhance health (Anciaes & Jones, 2022). The traditional Arabic souq combines both automobile and pedestrian traffic.

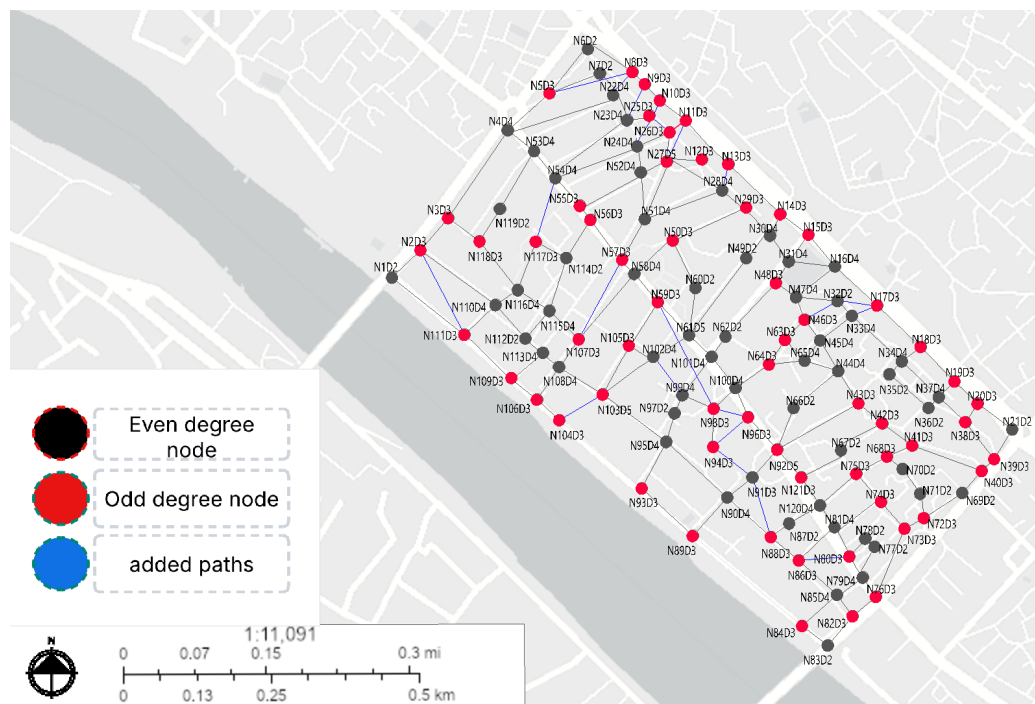


Figure 6. The valency of chosen nodes or degrees within the study area, where red nodes display the odd degrees, the black nodes display the even degrees, and the new added paths are in blue to achieve Eulerian (source: the authors).

4. Results

The results are organized into three main sections: identifying Eulerian paths, conducting space syntax analysis, and performing an in-field assessment. Firstly, identifying Eulerian paths is essential for connecting the entire urban network and guiding pedestrian circulation without repeated routes. Moreover, it ensures that commuters pass through each path once and only once. To evaluate its effectiveness, a space syntax analysis is carried out to compare the Arabic souq area before and after the network modifications required to establish the Eulerian path. Lastly, an on-site traffic assessment is conducted to demonstrate how the Eulerian configuration enhanced the integration of the primary streets, thereby improving access to historic and traditional architectural landmarks.

4.1 Identification of Eulerian

Results showed that the traditional Arabic souq area map graph does not contain an Eulerian path due to odd-degree nodes. To obtain Eulerian we converted the graph vertices from odd degree to even degree by adding and eliminating path segments. The resulting graph showed the following:

- 1) The Eulerian path joined four internal circuits, starting and ending at the red node labelled (N103D5), and is illustrated as a sequence of mathematical coding, as shown in Figure 7.
- 2) The Eulerian tour is based on visiting all edges in the graph once and only once, allowing the best pedestrian development coverage of all routes with no repetition.

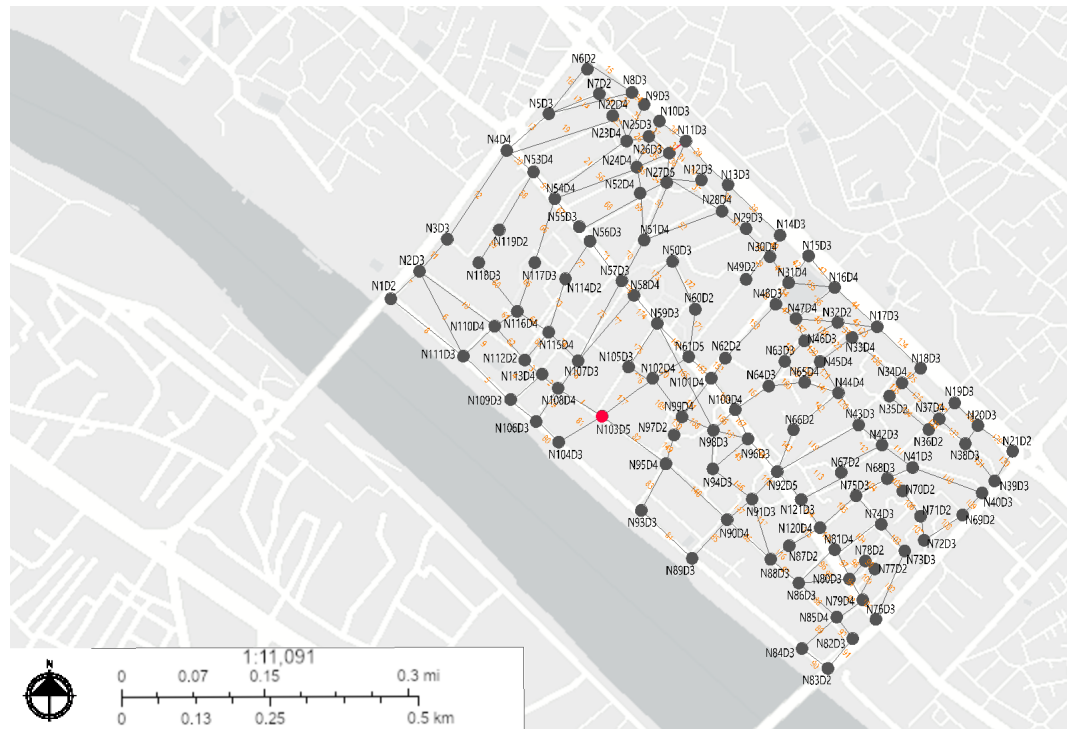


Figure 7. Eulerian, labelled with orange numbers, are graphed after editing paths by adding and eliminating those paths related to odd degree nodes. (source: the authors)

4.2 Space Syntax

Space syntax analysis was conducted on the current urban network of the Arabic souq (before Eulerian) by transforming the drawing map into a segment map. The spatial indices, including integration, connectivity, choice, and depth, were calculated within radii of 300 and 500 meters. These measures were weighted by angular connectivity to account for the influence of directional changes on spatial legibility and navigational ease.

The space syntax analysis presented in Figure 8 revealed a detailed understanding of the spatial configuration of the Arabic souq network by highlighting its structural and navigational properties. At a global scale (using a radius of 500 meters), the integration map showed that Al-Rasheed and Khan Marjan Street segments, coloured red, showed high levels of accessibility from all other points within the study area. The lower values, coloured blue, were located on the end of Al-Rasheed Street which lowers the ability to access the rich architectural details of the traditional façades .

The total angular connectivity map at the same radius reinforced this interpretation by identifying those same segments as active connectors, which supported both access and the whole structure of the spatial system. When the analysis was localized to a smaller radius of 300 meters, shifts began to appear in the integration patterns. Some segments that were not prominent in the global map become more integrated locally, indicating localized sub-networks with higher pedestrian accessibility. This change revealed variations in segment functionality depending on the spatial scale considered.

The choice analysis, weighted by angular connectivity, added another dimension by assigning segments that frequently are part of the shortest paths through the network. These paths, displayed in red, emerged as likely thoroughfares for both pedestrians and vehicles. They may not be the most directly accessible, but they play a crucial role in facilitating movement across the broader network. In contrast, the total depth metric, which considers the angular distance from each segment to all others, paints a picture of spatial hierarchy. The segments in blue are deep and relatively segregated, indicating their limited reach and possible use as low-traffic zones. The intelligibility scatter plot showed the local connectivity against global integration. It suggested that the network is legible where people can reasonably predict the overall structure based on their immediate surroundings. This relationship is essential for urban navigation, especially in walkable zones, as it enables a pedestrian to intuitively grasp their location within the alleyways. High average values for integration and choice affirmed the presence of highly connected and accessible segments, while depth metrics quantified the extent of spatial segregation in some parts of the network.

Space syntax analysis was re-applied to the Arabic souq area following the Eulerian transformation to assess its impact on the structure of the urban network as shown in Figure 9. The updated space syntax analysis revealed a different configuration in terms of spatial coherence and navigational flow. The Eulerian transformation involved the adjustment of the structure to ensure each segment is traversed more evenly or with balanced flows by improving connectivity. When the integration map was set to $R=500$, the red coloured segments are more evenly distributed compared to the pre-Eulerian state. This indicated a more uniformly integrated network, where accessibility from each segment to the rest of the system is enhanced. Similarly, the total angular connectivity map for $R=500$ is entirely red coloured, which suggests an increase in directness and continuity of movement across the souq's structure.

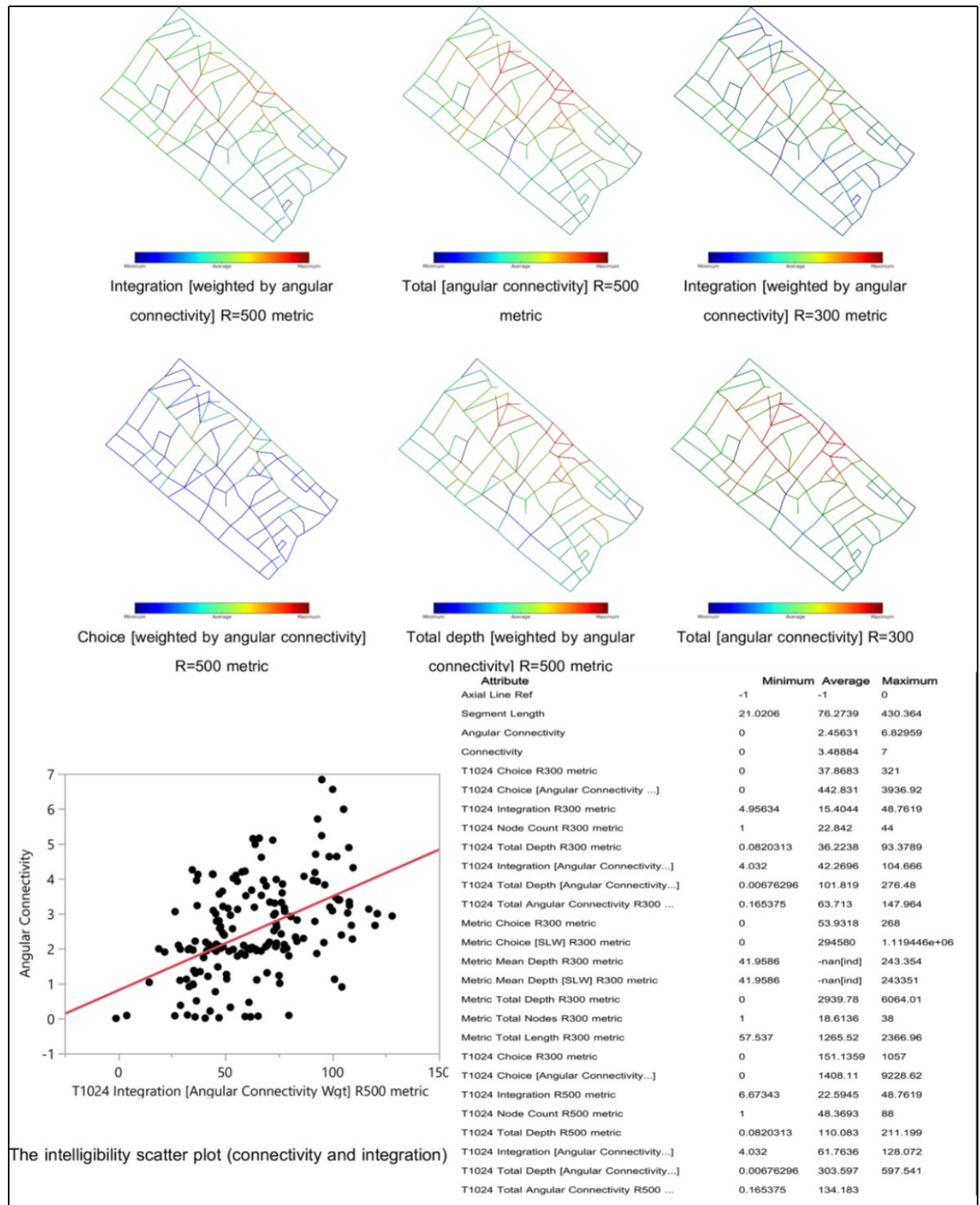


Figure 8. The attribute summary values and intelligibility of the axial map of the traditional Arabic souq which represent the correlation between integration and connectivity values before applying the Eulerian path (source: authors).

The directional efficiency has improved due to better alignment between the main pathways. The integration at R=300 reflected more local accessibility, fostered the importance of the central segments as primary areas of access, and showed smoother transitions into adjacent streets. Hence, the Eulerian transformation not only maintained but also refined local spatial intelligibility, making secondary paths more functionally integrated with main axes to support walkable and pedestrian friendly environment.

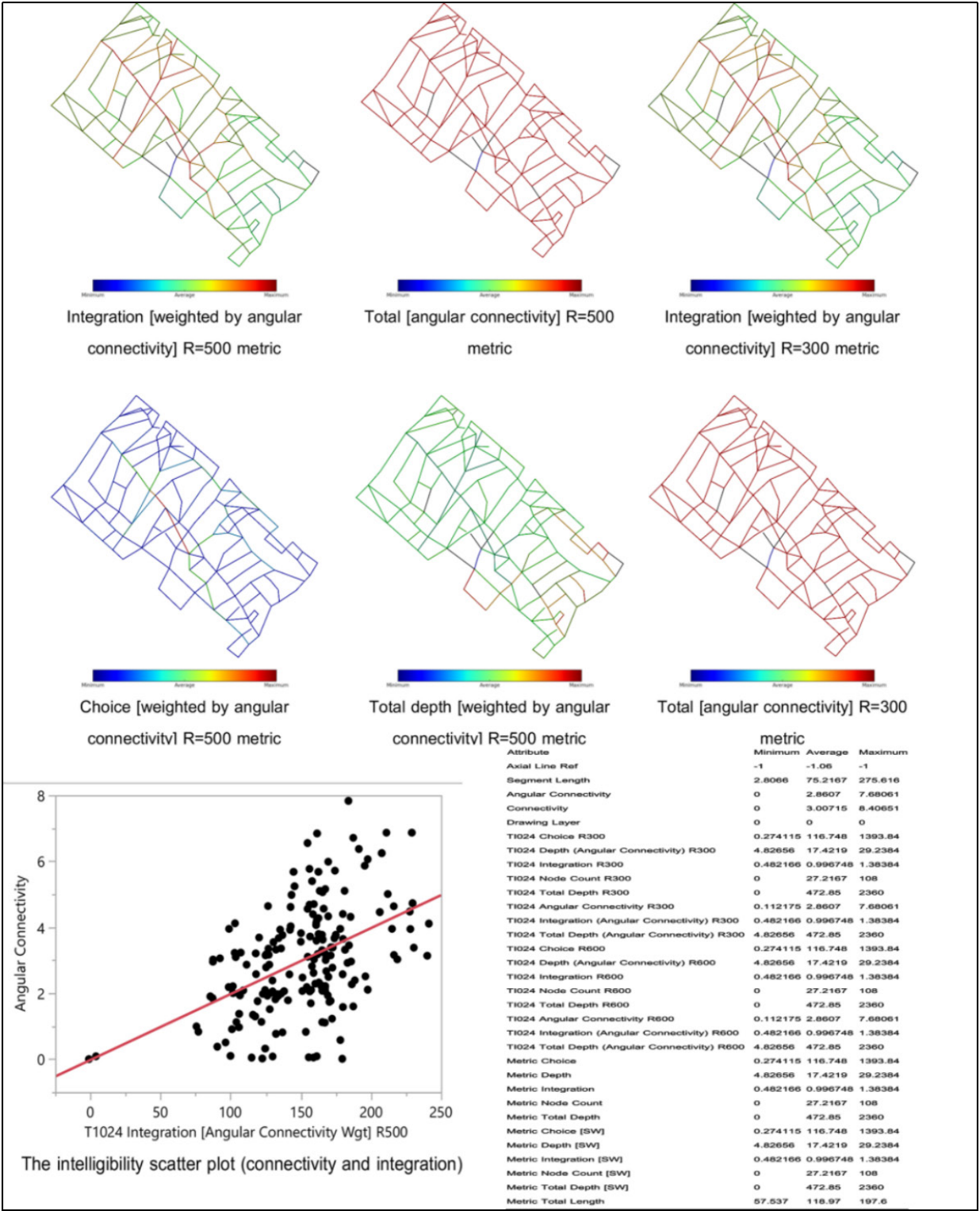


Figure 9. The attribute summary values and intelligibility of the axial map of the traditional Arabic souq which represent the correlation between integration and connectivity values after applying the Eulerian path (source: the authors).

The choice map at $R=500$ indicated through-movement potential, remaining relatively subdued in terms of variation (blue to green range), which suggested that although direct accessibility improved, the transformation may have flattened the hierarchy of movement. This enhanced the distribution of pedestrian traffic more evenly by reducing bottlenecks.

The total depth of the newly transformed system was more integrated overall, reflecting enhanced spatial accessibility throughout the configuration. Most of the network now lies within a mid-range angular depth from other segments, suggesting an improved spatial cohesion. The intelligibility scatter plot displayed a horizontal line of data which indicated the regularization of the network. The Eulerian-transformed Arabic souq showed a network that is more uniformly connected and integrated. This transformation promoted equitable access and simplified the navigation.

4.3 In-field Traffic

The in-field traffic assessment revealed that the main streets on the periphery of the study area are highly congested between 8:00 AM and 12:00 PM on weekdays, with peak hours at 12:00 PM. The combination of automobile and pedestrian circulation modes negatively impacts the safety of the traditional souq route system. After the application of Eulerian to the study area, the integrated paths expanded to include more traditional and historic buildings which allows a higher accessibility to the pedestrian as illustrated in Figure 10.

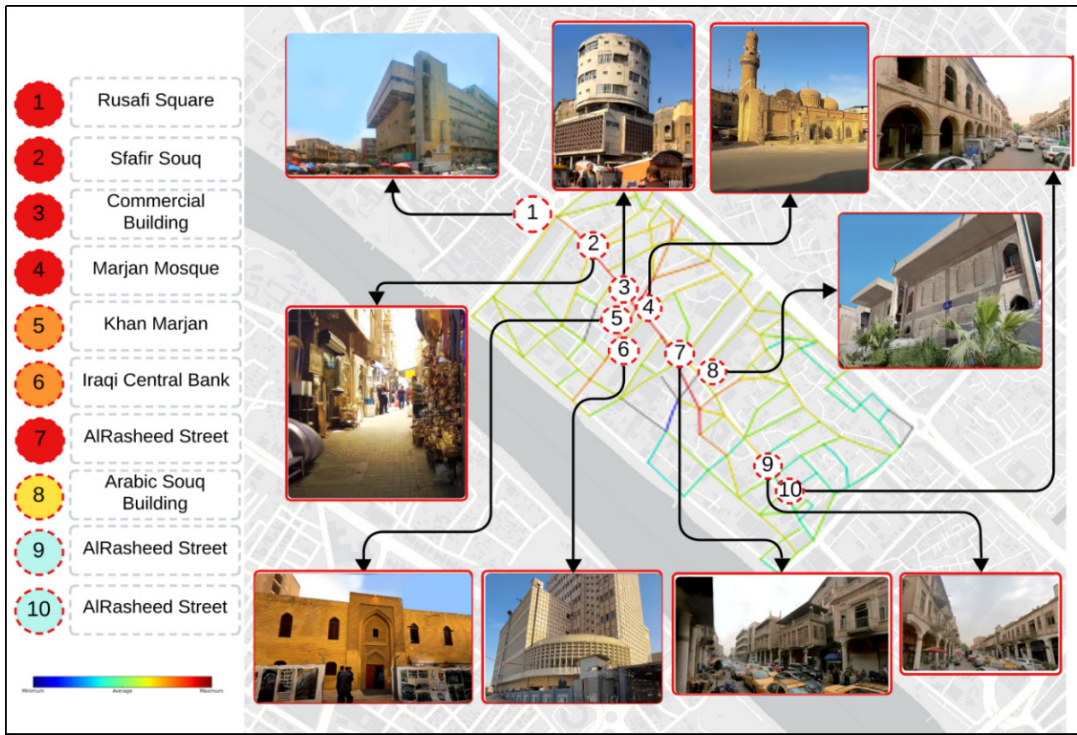


Figure 10. The traditional Arabic souq network (Eulerian application) shows how it fosters historic and traditional paths (source: the authors).

5. Conclusion

This research underscores the interdisciplinary significance of mathematical coding, particularly graph theory, in enhancing urban pedestrian development. Originally formalized by Euler, graph theory provides a structured approach to modeling movement networks, making it a valuable tool for urban analysis and planning. The case study of the traditional Arabic souq in Baghdad highlights the profound cultural, commercial, and historical importance of these spaces, necessitating sustainable urban interventions to revitalize pedestrian circulation. This study contributes to the existing literature by applying mathematical principles to the specific context of the traditional Arabic souqs. In addition, it is based on theoretical foundations in graph theory and space syntax. The study extends previous research through its integrated methodology, context-specific application, and emphasis on cultural preservation that supports sustainability. The findings align with previous literature regarding the structural applications of graph theory, the methodological framework of space syntax, and the goals of sustainable urban development. The novelty of this study lies in its specific focus on traditional Arabic souqs and its integration of multiple methodologies. The interdisciplinary approach and practical guidelines offer a significant advancement in the application of mathematical principles to complex urban challenges.

The application of Eulerian paths within the souq demonstrates how mathematical coding can optimize pedestrian routes, ensuring that all pathways are covered efficiently without redundancy. This process required converting odd-degree vertices to even degrees, enabling the modification of path networks to create a seamless and continuous pedestrian system. This study has demonstrated the critical role of computational methods in pedestrian planning, with Fleury's algorithm proving instrumental in defining Eulerian tours and optimizing historic circulation patterns. The integration of graph-theoretic transformations with space syntax analysis has provided quantitative insights into the impact of these modifications, revealing both the constraints and opportunities inherent in reconfiguring pedestrian networks. The results indicated an introduction of new or excluding other paths to achieve an Eulerian structure within the traditional Arabic souq marked a transformation of the space functions and how they are experienced. In the context of the Arabic souq area where narrow alleys and a deep-rooted cultural identity existed, the creation of additional routes is an effort to enhance movement while regarding the character of the historic fabric. These newly added paths completed the urban network and allowed the continuous pedestrian circulation without the need to retrace steps. Rather than disrupting the traditional layout, the new paths subtly bridged gaps between disconnected or isolated segments of the Arabic souq network.

Space syntax analysis revealed an improved spatial integration that resulted from the Eulerian application to the urban network. The new paths contributed to both global and local accessibility by transforming the souq into a more legible environment without compromising its intricate spatial logic. The improved connectivity ensured that important destinations like the historic and traditional buildings are more easily reached which supported the relationship between functional access and cultural significance. The study suggests that the newly added paths do not introduce imposed geometries; instead, they present a well-designed network that connects the existing traditional urban network, preserving the original layout. Given that most of the Arabic souq area land uses are public, integrating these new connections into the historic fabric became a more feasible process.

The in-field observations validated the theoretical predictions, ensuring that the mathematical model aligns with actual pedestrian behaviour. The broader implications of this study emphasize that graph theory and mathematical coding can mitigate spatial fragmentation, restore walkability, and promote social cohesion in historic urban environments. The integration of Eulerian logic into planning strategies fosters sustainable,

pedestrian-centric urbanism, countering the vehicular-dominated transformations that often disrupt traditional spatial order.

If we are to develop intelligible and navigable cities, we must acknowledge that urban spaces are not merely assemblages of buildings and streets, but dynamic configurational systems that govern human movement and interaction. Space is the structural medium through which societies organize themselves, and by leveraging its syntactical logic, we can enhance urban liveability:

1. To effectively apply these principles, urban policymakers and designers must embed graph theory algorithms into urban master planning. A fundamental shortcoming in contemporary urban planning is the failure to recognize street networks as relational systems, rather than isolated segments. Eulerian path analysis provides a rigorous mathematical basis for achieving continuous, uninterrupted movement, ensuring that pedestrian pathways are not residual spaces, but structurally integrated components of the urban fabric.

2. Prioritizing the spatial economy of pedestrian movement walkability is not merely about providing pedestrian infrastructure to ensure that the spatial structure itself generates movement. The most successful pedestrian environments emerge when urban grids are well-integrated, exhibiting high degrees of connectivity and choice. This requires restricting vehicular dominance in historically dense areas to prevent fragmentation of pedestrian flows and designing pedestrian routes with high global integration to ensure they form a central part of the urban movement economy, rather than peripheral, underutilized spaces.

3. Optimize pedestrian accessibility by predicting the impact of spatial interventions before implementation.

4. Reconfigure historic networks for contemporary use. The decline of traditional marketplaces often is a direct consequence of the disruption of their original movement networks. To restore their urban function, we must reinstate intelligible spatial hierarchies that promote natural pedestrian flows.

5. Deploy smart wayfinding technologies that work with the syntactic structure of space, rather than imposing artificial navigation aids. If we accept that the spatial form of a city generates its own dynamics, then urban design must evolve beyond piecemeal interventions and toward structural spatial thinking.

This mathematical coding application contributes to urban revitalization, supporting both economic activity and cultural engagement that fosters sustainability. Moreover, the coding underscores the importance of quantitative spatial analysis in shaping urban decision making, reinforcing the need for a systematic, computational approach in sustainable pedestrian planning.

6. Recommendations and Future Works

- 1) The study recommends that planners, urban designers, and scholars conduct multi-method analyses and compare theoretical outputs with real-life status to obtain reliable results. Policymakers should foster pedestrian paths within cities.

- 2) Future work can apply heuristic algorithms to graphs of urban development in studying urban structure characteristics.

- 3) Applying machine learning algorithms to further optimize urban mobility networks based on real-time data.

- 4) Investigate the application of graph theory in multimodal transport integration, connecting pedestrian pathways with public transit.

- 5) Conduct comparative studies applying Eulerian paths in other historic urban environments beyond the Arabic souq.

Human Subjects

No human subjects are employed in this study.

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Data Availability

All data used in this study are included in the body text.

Use of AI

During the preparation of this manuscript, the authors used DeepSeek to improve readability and language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

Author Contributions

Tamara Mutaz: Conceptualisation, Methodology, Writing the Original Draft, Investigation, Visualization.

Zainab Khalid: Conceptualisation, Supervision, Writing - Review & Editing.

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