A Mathematics Instructional Model by Integrating Problem-Based Learning and Collaborative Learning Approaches

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Abstract

According to several researches conducted by several organizations, such as Ministry of Education (MOE), Institute for the Promotion of Teaching Science and Technology (IPST), and National Institute of Educational Testing Service (NIETS), it was found that most Thai students did not succeed in learning mathematics in terms of content and mathematics processes. This research results were consistent with the facts and findings of several surveys which were conducted based on the situation happening in real life. Therefore, problems in learning mathematics could be concluded in 3 significant points, including (1) mathematical problem solving ability, (2) mathematical communication ability, and (3) mathematical connection ability. The purposes of this research included (1) to develop a mathematics instructional model by integrating problem-based learning approach and collaborative learning approach to enhance mathematical problem solving, communication, and connection abilities of sixth grade students, and (2) to study the effects of the use of the instructional model.

The research procedure included two steps. The first step was the development of instructional model and the second step was the experiment of the developed instructional model in classrooms. Integrating problem-based learning approach and collaborative learning approach and also analyzing and synthesizing the related concepts, theories, and researches were brought to develop the model. The instructional model was further experimented with the sample group of sixth grade students. The samples

consisted of two classrooms, one experimental group and one control group, in which there were 24 students each, at Wat Taranaram School under the Suratthani Primary Educational Service Area Office 2. The experimental period covered 38 hours conducted by using the research tools, including the lesson plans under the developed model, conventional lesson plans, as well as tests for mathematical problem-solving, communication, and connection abilities. Quantitative data were analyzed by using means of arithmetic mean ($\bar{X}$), standard deviation (S.D.) and t-test.The research results could be summarized as follows:-

1. The developed instructional model included four elements which are (1) principles, (2) objectives, (3) steps of instructional process, and (4) learning assessment and evaluation. In addition, the instructional process consisted of four steps, including (1) encouraging students' attention in encountering challenging problems, (2) practicing enthusiastically for searching knowledge, (3) collaboratively examining their knowledge and concluding by group consensus, and (4) applying the knowledge.

2. The developed instructional model was efficient and could be used to enhance students' mathematical problem solving, communication, and connection abilities as follows:

2.1 Mathematical problem solving, communication, and connection abilities at the post-learning stage of the experimental group were significantly higher than those of the control group at the significant level of .05; and

2.2 Mathematical problem solving, communication, and connection abilities of the experimental group at the post-learning stage were significantly higher than those abilities at the pre-learning stage at the significant level of .05.

Key Words: Collaborative Learning; Problem-Based Learning; Mathematical Problem Solving ability; Mathematical Communication ability; Mathematical Connection ability; Instructional Model

Introduction

The society in the era of globalization is full of economic, political, as well as science and technological competitions. The countries, which do not foster the national development, will not be able catch up with the rest of the world. One of the significant recommendations for national development is to provide alternative channels to access resources of knowledge and information, as well as encourage the social participation in all levels. Education is a fundamental base for this aforementioned development.

In terms of human resources development, the Human Achievement Index (HAI) was used to assess the efficiency of human resources development. There were 8 compositions of this index, including (1) health, (2) education, (3) work life, (4) income, (5) residence and environment, (6) family and community, (7) transportation and communication, as well as (8) social participation (UNDP, 2007). In terms of education, the HAI assessment result indicated that Thailand must increase the average years of education per capita to 10 years and the education must focus on building student's knowledge and skills in mathematics and language learning (NESDB, 2006).

From several researches conducted by various organizations, it was found that the majority of Thai students did not succeed in studying mathematics, in particular in the areas of content and thinking processes (Ministry of Education,2009; IPST,2008 and NIETS, 2008). This research results were consistent with the facts and findings of several surveys which were conducted based on the real life situations. Regarding the mentioned researches and survey results, the Thai students' problems in learning mathematics could be divided in three areas as follows:-

1) Mathematical problem solving ability: the majority of students could not effectively understand and analyze mathematics problems due to the lack of knowledge and skill in figuring out appropriate solutions to solve the problems. Therefore, they were unable to figure a correct answer or reversely analyze an answer back to a problem. This was consistent with Lynn (1993) who found that a reason why students could not solve mathematical problems was the lack of experience in various methods of

problem solving. Therefore, the problem solving skills had to start from a skill to abruptly find methods or solutions to solve complicated problems which depends on individual intelligence (Polya, 1980) and basic knowledge which could be appropriately applied to solve unfamiliar problems (Krulik and Rudnick, 1993). In addition, Wilson (1993) mentioned that problem solving process was a dynamic process which could be implemented step-by-step and with flexibility. Lynn (1993) also found that factors impacting problem solving behaviors included group collaboration, social control, and social rules.

2) Mathematical communication ability: the majority of students could not write or represent their ideas by using appropriate language, mathematics symbols, and counting pictures. In addition, they were unable to interpret mathematics language by using the common language to effective communicate their thoughts and opinions as well as exchange the information and knowledge on mathematics with others (NCTM, 1989; Tomas, 1991). Therefore, Rowan and Morrow (1993) proposed an instructional guideline that teachers were recommended to use tangible examples to encourage students to describe things they found out, use content which is close to students' lifestyle, use open-ended questions to allow students to communicate their thoughts, and use the collaborative learning approach. According to Johanning's research (2000), it was found that descriptive writing was one of many ways to encourage students to learn mathematics. Once students could communicate their thoughts to others by writing on the paper, they would gain more confidence in sharing opinions when collaborating with a group and be more eager to think and participate in mathematical learning process.

3) Mathematical Connection ability: The majority of students could not link their existed knowledge into mathematics content, as well as could not link mathematical content into other subject areas and their real life (Coxford, 1995; Kennedy and Tipps, 1994). In addition, Makanong (2004) mentioned that the mathematical connection was learners' abilities to link their mathematics knowledge and problems gained from classes to the current problem or situation with which they were dealing. NCTM (2000)

explained the direction of mathematical connection as the Chaos Theory, in which the connection among real-life situations, the relation of mathematics-related subjects which are different from existed teaching subjects, and the relation of mathematical contents themselves are integrated.

This paper is intended to exhibit some parts of the aforementioned research by focusing on the development of a mathematical instructional model by integrating problem-based learning approach and collaborative learning approach to enhance mathematical problem-solving, communication, and connection abilities of sixth grade students.

Objectives

1) To develop instructional model by integrating problem-based learning approach and collaborative learning approach to enhance mathematical problem solving, communication, and connection abilities of sixth grade students;

2) To study the effects of the use of the instructional model developed by comparing the pre- and post-learning of an experimental group, as well as comparing an experimental group and a control group.

Literature Review

This research was conducted based on two significant principles, including problem-based learning (PBL) and collaborative learning (CL), as following details.

1. Problem-based Learning (PBL)

Tan (2003: 30-31) mentioned 10 key thoughts of problem-based learning (PBL) as follows:-

1) Problem is the start of learning process;

2) Existing real-life problems are complicated and can be developed into the real problem;

3) There are various perspectives to view each problem, and we need to use various fields of knowledge to solve it;

4) Challenging problem focuses on determining learning object and learning that object more;

5) Students' self-responsibility can be developed by leading themselves to learn by acquiring various kinds of information;

6) There are various forms and places of learning resources which are related to learning process;

7) PBL is collaborative learning which includes communication, cooperation, and work in a small group of students. Students' ability can be developed by having high interaction between friends and presenting their ideas to the group;

8) The development of problem examining and solving ability is the center of knowledge apart from knowing knowledge enough to solve problems. Therefore, instructors should solely play role as a facilitator and advisor by raising questions to promote better understanding;

9) Learning should be concluded by synthesizing knowledge based on problem, then thoroughly integrated that knowledge to reflect ideas and review it further; and

10) Learning should be also concluded based on problem by conducting evaluation and reviewing learners' experiences and learning processes.

Regarding Tan's thoughts on problem-solving learning, it could be concluded that problem was the start of learning processes when it existed in the real-life and was complicated as well as related to many fields of study. Problems challenged learners to determine what they wanted to learn and led them to acquire knowledge from various resources in order to further apply in solving problems. Students should learn collaboratively in a small group while teachers were recommended to solely guide and facilitate the learning.

In addition, the Medical School at McMaster University in Canada (2010) also indicated that the learning processes should be driven by problems, and learners must solve problems by themselves and by collaborate with others in a group. Problems should be highlighted to encourage students to collaboratively figure out and conclude appropriate solving methods. Teachers should be a guide and a facilitator during the learning processes. The Medical School at McMaster University in Canada also proposed the

five characteristics of PBL as follows:-

1) Learning must be driven by open-ended questions which are challenging and flexible to answer;

2) Problem must be based on particular context;

3) Students must eagerly conduct the searching and solving process in a small group of 5 people by themselves;

4) The key problems must be identified and there must be a consensus on problem-solving approaches which will be further implemented; and

5) Teachers play an important role in learning by advising learning guideline and promoting learning- and searching-friendly environment.

Furthermore, Savery (2011 : Online); Arends (2009 : 387); Howard (2003); Illinois Mathematics and Science Acadamy (2003); Savin (2000 : 17-18); Barrow (1996 : 5-6); and Savoie and Hunges (1998 : 73) agreed on the characteristics of PBL that it was necessary to be a challenging and complicated problem which content must be related to learners' existed experiences and must come from the integration of various subject areas. Problem must encourage learners to solve, and learners must have responsibility in self-learning. There were several approaches to solve a problem, such as communicating and interacting among classmates by having a teacher as a learning facilitator.

In addition, Hmelo and Evenson (2000 : 4); Torp and Sage (1998 : 15); and Gijselaers (1996 : 13-14) consistently mentioned that PBL was related to learning theory, called constructivism, which was rooted from learning theory of Piaget and Vygotsky who believed that learning was a development process of intelligence which learners could develop this knowledge by themselves. The learning development process took place from the interaction between learners and environment, which led to the absorption of new experiences, and the adaptation of intelligence to the new environment.

In this research, the PBL characteristics could be summed up that students' learning mainly occurred through problem learning. Student could learn new knowledge from problem solving process by relating their

existing knowledge with problematic situations and by acquiring new relevant knowledge to solve problems. The solutions for each problem could be various.

2. Collaborative Learning (CL)

Lara and Brown (2011 : Online) compared this principle with the umbrella circle which had various forms of collaborative learning rooted from small group project. Arends (1994) mentioned that CL was the teaching process which groups students to collaborate in small groups, and everyone had joint responsibility with the same goal. They must also exchange views and create good relationship among one another. MacGregor and Smith (2011) mentioned that CL was the collaborative educational process among related people. This also included the collaboration among a group of students and teachers, in which there were not less than 2 people involved to study research, trying to understand the content's meaning, and delivering something. This must focus on increasing students' role in presenting knowledge to the class; while switching teacher's role in conducting learning activities to designing learning activities and encouraging students to apply their knowledge.

Swan (2006) mentioned 6 key ideas of collaborative learning (CL) as follows:-

1) CL leads to the creation of student's knowledge, this means knowledge arising from the development of formative evaluation by applying appropriate techniques and improving teaching approaches based on student's abilities and needs;

2) CL brings ambiguous issues to the discussion by organizing learning activities to exchange views on any debatable issues. The mentioned activities must be open to encounter and concluded by making consensus decisions based on results from the debate;

3) CL uses high-level questions which can help promote thorough description, analysis, and future application;

4) CL encourages the small group collaboration which helps encourage thinking and analyzing process, as well as positive discussion instead of arguments. The most important thing is to have a joint responsibility

on the same goal among the small group members;

5) CL helps promote the provision of reasons instead of solely focusing on the answer correction. This is a deep goal of learning although students usually focus on what they have done and what they have learned in the past;

6) CL promotes various tasks which require the collaboration to achieve. This tasks can be generally found and further extended and they should be able to encourage the decision making, descriptive explanation, and creative thinking by using questions concerning "if… what" and "if… what not,";

7) CL creates connections between related subject matters. In general, it is difficult for students to link each learning subject content to other particular contexts so that teachers play an important role in facilitating students to linking their knowledge; and

8) Technology is another alternative approach to promote CL. Interactive computer program can be used as a whiteboard to encourage students to learn by watching attractive motion pictures.

Regarding Swan's principle, CL characteristics could be concluded that CL took place based on ability of students in a small group which had responsibility on the same goal. The group discussed on some ambiguous issues, and the discussion further led them to the explanation among group members by using questions and expressing reasons in order to make other group members agree on the same thing.

In this research, it was concluded that CL took place naturally in a small group of random students, including high ability, moderate ability, and low ability, when there were group explanation and discussion in order to solve problems. Group members used reasons to explain their ideas and discuss with others to make everyone agree on the same thing and has responsibility on the same goal.

Conceptual Framework

1. Problem-based Learning (PBL) Principle

PBL means challenging and real-life-related problems which

encourage learners to pay attention on knowledge acquisition from various resources by relating the problems to problematic situation, as well as collaborating in solving the problems and exchanging views. There are 3 principles of PBL, including (1) the creation of challenging or real-life-related problems which encourage learners to acquire knowledge by relating knowledge to the problems, (2) learners' collaboration in analyzing problematic situation, eagerly acquiring knowledge, and using knowledge to solve the problems, and further apply, as well as (3) the communication during collaborative processes to exchange knowledge and views on various problem-solving methods.

2. Collaborative Learning (CL) Principle

CL means a naturally learning process by letting learners, possessing different characteristics and levels of knowledge, collaborate in learning and reasonably discuss to reflect their ideas on problem solving methods and create deliverables. There are 4 principles of CL, including (1) grouping learners based on the different levels of abilities, (2) letting learners naturally collaborate in learning by arguing with reasons, (3) facilitating the discussion among learners in order to solve the ambiguous matters, and (4) creating mutual understanding among learners to build up the deliverables on which they have joint responsibility.

3. Principles rooted from the Integration of PBL and CL

There are 4 principles of PBL and CL Integration, including (1) creating challenging or real-life-related problems, (2) grouping learners into 3 groups based on the different levels of abilities (high ability, moderate ability, and low ability at the ratio of 1:1:2) to determine their learning needs, (3) naturally collaborating in learning, and (4) communicating during learning process by exchanging ideas on problem solving methods.

According to the analysis and synthesis of the results of studies regarding the principles and characteristics of PBL and CL, the researcher integrated both principles to develop steps of a mathematics instructional model which helped enhance students' ability in problem solving, communicating, and mathematics connecting, The conceptual framework of this research could be depicted as the following diagram.

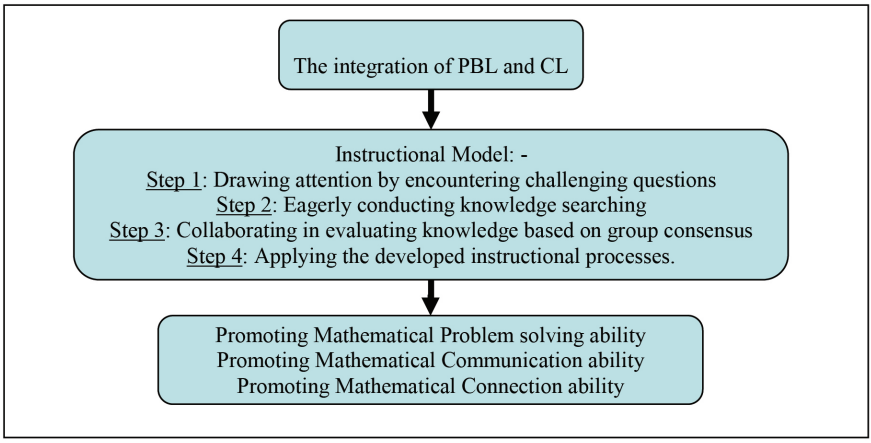

Figure 1 Exhibits the research conceptual framework

Methodology

The research methodology on the development of a mathematics instructional model by integrating problem-based learning and collaborative learning approaches to enhance mathematical problem solving, communication, and connection abilities of sixth grade students included three implementation steps as follows: -

Step 1: Developed a mathematics instructional model which includes 3 processes, including (1) study needs and problematic situation, (2) study thoughts, theories, and researches regarding instructional model development, and (3) develop, test, and conduct an experiment on the appropriate mathematical instructional model.

Step 2: Prepared the experiment of a developed model by developing tools and testing the quality of each tool as follows;

2.1 Created 6 parallel tests and their scoring criteria. The parallel tests included pre-learning and post-learning tests which had similar structure. However, as this research aimed to promote mathematical problem solving, communication, and connection ability and did not aim to necessarily evaluate the learned knowledge to represent such ability, the content of such parallel tests included (1) the same content as student learned in the class (2) If content was different, it should have been learned based

on the content sequence or from the lower level classes. The details of the mentioned tests were as follows:-

- Test 1 Pre-learning Test for Mathematical Problem Solving Ability: 2 questions and 12 points each;
- Test 2 Pre-learning Test for Mathematical Communication Ability: 3 questions and 9 points each;
- Test 3 Pre-learning Test for Mathematical Connection Ability: 3 questions and 6 points each;
- Test 4 Post-learning Test for Mathematical Problem Solving Ability: 2 questions and 12 points each;
- Test 5 Post-learning Test for Mathematical Communication Ability: 3 questions and 9 points each;
- Test 6 Post-learning Test for Mathematical Connection Ability: 3 questions and 6 points each;

2.2 Proposed the aforementioned 6 test and scoring criteria to the experts to assess their quality. The IOC values were equal to 0.84, 0.83, 0.87, 0.88, 0.87, and 093 respectively;

2.3 Tested students outside the sample group with the revised tests and scoring criteria twice and checked their points to test the tests' quality. In this connection, the selection of difficulty had to be valued between 0.2-0.8 and the discrimination value had to be not less than 0.2 in order to deem the test validity. The details were as follows:-

- The first quality test of the tools: Used the pre-learning tests with 24 students at Ta-Khun School and used the post-learning tests with 40 students at Punpin Pittayakom School;
- There were 3 questions for the Pre-learning Test for Mathematical Problem Solving Ability: the difficulty value was between 0.43-0.46 and the discrimination value was between 0.52-0.55;
- There were 5 questions for the Pre-learning Test for Mathematical Communication Ability: the difficulty value was between 0.31-0.50 and the discrimination value was between 0.38-0.75;
- There were 5 questions for the Pre-learning Test for Mathematical Connection Ability: the difficulty value was between 0.40-0.52 and the

discrimination value was between 0.67-0.75;

- There were 3 questions for the Post-learning Test for Mathematical Problem Solving Ability: the difficulty value was between 0.28-0.51 and the discrimination value was between 0.31-0.55;
- There were 5 questions for the Post-learning Test for Mathematical Communication Ability: the difficulty value was between 0.35-0.40 and the discrimination value was between 0.43-0.58; and
- There were 5 questions for the Post-learning Test for Mathematical Connection Ability: the difficulty value was between 0.39-0.55 and the discrimination value was between 0.35-0.55; and
- For the second quality test of the tools, the Researcher deleted questions with low discrimination value based on the analysis results from the first test of the tools, and then used the revised pre-learning tests with 27 students at Ban Tham Niab School and used the revised post-learning tests with 43 students at Chaiya Wittaya School;
- There were 2 questions for the Pre-learning Test for Mathematical Problem Solving Ability: the difficulty value was between 0.50-0.59 and the discrimination value was between 0.57-0.69;
- There were 3 questions for the Pre-learning Test for Mathematical Communication Ability: the difficulty value was between 0.32-0.51 and the discrimination value was between 0.21-0.54;
- There were 3 questions for the Pre-learning Test for Mathematical Connection Ability: the difficulty value was between 0.36-0.63 and the discrimination value was between 0.54-0.57;
- There were 2 questions for the Post-learning Test for Mathematical Problem Solving Ability: the difficulty value was between 0.37-0.47 and the discrimination value was between 0.33-0.65;
- There were 3 questions for the Post-learning Test for Mathematical Communication Ability: the difficulty value was between 0.37-0.44 and the discrimination value was between 0.39-0.51; and
- There were 3 questions for the Post-learning Test for Mathematical Connection Ability: the difficulty value was between 0.41-0.45 and the discrimination value was between 0.35-0.61; and

2.4 Used the aforementioned tests and their effective scoring criteria with the sample group of students.

Step 3: Conduct an experiment on the application of a developed model in order to evaluate the quality of the model.

For this research, the Researcher conducted an experiment of a developed instructional model with the sample group of sixth grade students. The samples included two classrooms, one experimental group and one control group, and 24 students for each classroom, at Wat Taranaram School under the Second Suratthani Primary Educational Service Area office 2. The experimental period was 38 hours. The Researcher taught both experimental and control group by herself.

Regarding the selection of both experimental and control group, for this research the Researcher selected classes to be experimental and control group as follows:-

1) Used results from the Mathemetical Assessment Test for the Academic Year B.E. 2554 of each class to calculate for their average, and then selected 2 classes which had close average score; and

2) Conducted variance test by using F-test in which it was found that the variance tests were indifferent, conducted the test for the difference of the average of mathematical assessment test results by using t-test in which it was also found the indifference, and then conducted the simple random to select such classes as experimental and control group.

Results

The research on the development of the instructional model by integrating problem-based learning approach and collaborative learning approach started from conducting a study on needs and problem situation, and then studied relevant thoughts, theories, and researches regarding possible solutions to solve problems in terms of mathematical problem-solving, communication, and connection abilities. After that, the researcher thoroughly analyzed the importance of this problem and linked it to the solutions to promote those three abilities of students, including problem-based learning (PBL) and collaborative learning (CL). The researcher also

studied ideas and researches in the areas of problem-based learning and collaborative learning which can promote problem-solving, communication, and connection abilities in mathematical learning in order to synthesize and develop a conceptual framework and key characteristics. Therefore, this led to the development, examination, and experimentation of the instructional model created by the researcher. This model was also evaluated by the appropriate experts to examine the consistency between conceptual framework and the model.

The evaluation of the developed model was conducted in 2 forms, including the examination of the instructional curriculum and the test on mathematical problem-solving, communication, and connection abilities which were provided with the scoring criteria. Once the researcher believed that the developed model can be effectively used, the model was further experimented with the sample group. The research results could be summarized into some significant points as follows:-

1. The developed instructional model consisted of 4 key elements as follows:-

1.1 Principles of the instructional model included the following characteristics:-

1) Creating challenging problems or problems related to everyday life in order to encourage students' interest in acquiring knowledge or finding solutions from various sources of information, leading themselves, and relating their existing knowledge with a problematic situation;

2) Dividing groups of students based on their various knowledge backgrounds, the same goal to be responsible for, and jointly collaboration in analyzing a problem by focusing what they wanted to learn and eagerly acquiring necessary knowledge to solve problem with various solutions;

3) Naturally learning together both in a group and across groups. The students were responsible for what they collaboratively learned, discussed, explained, and presented with reasons on challenging issues, argued to create better understanding on any matter, and utilized knowledge to solve a problem or to finish an assignment;

4) Communicating and exchange of knowledge to gain ideas and guidelines to find an answer or solve a problem.

1.2 Objectives of the instructional model: This development of the instructional model aimed to promote the mathematical problem solving, communication, and connection ability.

1.3 Learning process of the developed instructional model included 4 steps as follows:-

1) Drawing attention by encountering challenging questions: -

a. Organizing mathematics-friendly environment and introducing informational resources for classroom and outside-classroom learning;

b. Using appropriate questions to encourage students to collaborate in finding answers;

c. Dividing students into 4 small groups by mixing students' ability level. There were high ability, moderate ability, and low ability at the ratio of 1:1:2, derived from the percentile of the mathematical assessment test for the academic year B.E. 2554; and

d. Presenting challenging problems or real life problems to encourage students to solve the problems.

2) Eagerly conducting knowledge searching

a. Encouraging students to analyze problem and knowledge to be used to solve problems.

b. Assigning students in each group to collaborate in setting the working target, analyzing, and discussing on problem issues in order to determine what they want to learn, which could include (1) subject areas, (2) principles, and (3) problem-solving processes; and

c. Assigning students in each group to eagerly and collaboratively conduct the searching process to seek for useful information which can be used to solve the problem.

3) Collaborating in evaluating knowledge based on group consensus

a. Assigning each group to collaboratively exchange views and thoughts by (1) speaking or writing ideas with mathematics language

and symbols such as charts, pictures, and symbolic sentences, (2) speaking or writing solutions based on mathematics principles, knowledge, and (3) systematically and completely presenting the thoughts and ideas; and

b. Letting each group select a representative to solutions to the class. Encouraging other students to discuss and reflect their opinions with reasons and then letting the class make decisions which could be various.

4) Applying the developed instructional processes

a. Providing other examples and situations so that students can practice to link their existed knowledge to solve the new problems and deliver a new task; and

b. Monitoring students to use knowledge and experiences to solve the problem and deliver a task.

1.4 Learning assessment and evaluation

For this research, the Researcher presented the developed instructional model by conducting a test through the use of learning management plan and testing for mathematical problem solving, communication, and connection ability of students, which included an experimental and a control group. The details were as follows:-

1. Conducted a pre-learning test with both experimental and control group by using the tests for mathematical problem solving, communication, and connection ability of students in accordance with contents learned earlier including contents for fifth and sixth grade students which were taught during the first semester in order to assess students' basic knowledge, such as the decimal number, square shape, triagle shape, circle shape, and their applications.

2. Conducted a post-learning test with both experimental and control group by using the parallel tests for mathematical problem solving, communication, and connection ability of students in accordance with contents for sixth grade students, such as the decimal number, square shape, circle shape, and their applications.

2. Regarding the effects of the use of the instructional model, there were significant findings as follows:-

2.1 Students who had been taught with the mathematical instructional model which is developed based on problem-based learning and collaborative learning approach after learning had higher level of mathematical problem-solving ability than those who had been taught with traditional model at the significant level of .05 as appeared in Table 1.

Table 1 The Comparison of the Average of Mathematical Problem Solving Ability after Being Experimented Between an Experimental Group and a Control Group (the Full Score was 24 Points)

		N	$\overline{X}$	**SD**	**P**
Experimental	Group	24	13.75	5.152	0.014*
Control	Group	24	10.83	3.608	

* $P < .05$

2.2 Students who had been taught with the mathematics instructional model which was developed based on problem-based learning and collaborative learning approach after learning had higher level of mathematical problem-solving ability than before being taught at the significant level of .05 as appeared in Table 2.

Table 2 The Comparison of the Average of Mathematical Problem Solving Ability after Being Experimented of a Control Group between pre and post learning (the Full Score was 24 Points)

	N	$\overline{X}$	**SD**	**P**
Pre-learning	24	9.46	4.606	.000*
Post-learning	24	13.75	5.152	

* $P < .05$

2.3 Students who had been taught with the mathematics instructional model which was developed based on problem-based learning and collaborative learning approaches after learning had higher level of

mathematical communication ability than those who had been taught with traditional model at the significant level of .05 as appeared in Table 3.

Table 3 The Comparison of the Average of Mathematical Communication Ability after Being Experimented Between an Experimental Group and a Control Group (the Full Score was 27 Points)

	N	$\overline{X}$	**SD**	**P**
Experimental Group	24	14.13	6.986	.020*
Control Group	24	10.42	5.053	

* P < .05

2.4 Students who had been taught with the mathematics instructional model which was developed based on problem-based learning and collaborative learning approach after learning had higher level of mathematical communication ability than before being taught at the significant level of .05 as appeared in Table 4.

Table 4 The Comparison of the Average of Mathematical Communication Ability after Being Experimented of a Control Group between pre and post learning (the Full Score was 27 Points)

	N	$\overline{X}$	**SD**	**P**
Pre-learning	24	10.00	3.600	.001*
Post-learning	24	14.13	6.986	

* P < .05

2.5 Students who had been taught with the mathematics instructional model which was developed based on problem-based learning and collaborative learning approach after learning had higher level of mathematical connection ability than those who had been taught with traditional at the significant level of .05 as appeared in Table 5.

Table 5 The Comparison of the Average of Mathematical Connection Ability after Being Experimented Between an Experimental Group and a Control Group (the Full Score was 24 Points)

	N	$\overline{X}$	SD	P
Experimental Group	24	9.75	4.406	.009*
Control Group	24	7.25	2.364	

* P < .05

2.6 Students who had been taught with the mathematics instructional model which was developed based on problem-based learning and collaborative learning approach after learning had higher level of mathematical connection ability than before being taught at the significant level of .05 as appeared in Table 6.

Table 6 The Comparison of the Average of Mathematical Connection Ability after Being Experimented of a Control Group between pre and post learning (the Full Score was 18 Points)

	N	$\overline{X}$	SD	P
Pre-learning	24	6.08	3.501	.000*
Post-learning	24	9.75	4.406	

* P < .05

Conclusions

This research had 2 objectives, including (1) to develop instructional model by integrating problem-based learning approach and collaborative learning approach to enhance mathematical problem solving, communication, and connection abilities of sixth grade students, and (2) to study the outcomes of the above-mentioned instructional model by comparing the pre- and post-learning of an experimental group, as well as comparing an experimental group and a control group. For the first objective, it was found that the

developed instructional model had 4 key elements, including (1) principle, (2) objective, (3) learning process of the developed instructional model, and (4) learning evaluation and assessment. There were 4 steps of learning process of the developed instructional model as follows:-

1) Drawing attention by encountering challenging questions;

2) Eagerly conducting knowledge searching;

3) Collaborating in evaluating knowledge based on group consensus; and

4) Applying the developed instructional processes.

For the second objective, it was found that (1) mathematical problem solving, communication, and connection ability at the post-learning stage of the experimental group was significantly higher than those of the control group at the significant level of .05, and (2) mathematical problem solving, communication, and connection ability of the experimental group at the post-learning stage was significantly higher than those at the pre-learning stage at the significant level of .05.

The outcome which teachers could gain from this research was the application of the mathematics instructional model developed by the integration of PBL and CL principles to an instruction either for mathematics or other subject areas, such as the creation of real life problems by integrating relevant subject areas, including Thai language, social studies, and science, for student in both elementary and secondary schools.

The outcome which students could gain from this research was knowledge and ability in solving mathematics problems, as well as communicating and connecting mathematics knowledge. This helped enhance students' knowledge and understanding in the mathematical concepts and processes, as well as helped entertain students, create social interaction, and enhance students' self-confidence in expressing their ideas and opinions.

Acknowledgements

This research was funded by the Chulalongkorn University Graduate School Thesis Grant.

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