



A STUDY OF STUDENTS' SYSTEMATIC THINKING DEVELOPMENT IN THE PROBABILITY LESSON THROUGH USING CONCRETE-PICTURE-ABSTRACT (CPA) COMBINED WITH GRADUAL RELEASE OF RESPONSIBILITY (GRR)

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Abstract

Systematic thinking is critical in mathematics as it helps students solve complex problems and develop a deep understanding of mathematical concepts. However, many students still struggle to develop these skills effectively. This study aimed to (1) compare students' systematic thinking in lessons on probability using Concrete-Picture-Abstract (CPA) teaching strategies combined with Gradual Release of Responsibility (GRR) and (2) compare the development of four aspects of systematic thinking of students using Concrete-Picture-Abstract teaching strategies (CPA) combined with Gradual Release of Responsibility (GRR). This research was conducted with a sample group of 40 Matthayom Suksa 3 (Grade 9) students, randomly selected using the cluster random sampling method. It employed pre-experimental research with pre- and post-tests. 8 learning management plans were used, incorporating the CPA technique along with GRR in the lesson on probability in research. A systematic thinking test was administered before and after the lesson, and a dependent t-test was used for data analysis. The study found that (1) students' systematic thinking significantly increased after the experiment at the .01 level, and (2) the scores of the four sub-skills of systematic thinking significantly increased after the experiment at the .01 level. These results indicate that the CPA approach combined with GRR effectively promotes systematic thinking.

Keywords: Systematic thinking, Concrete-picture-abstract (CPA), Gradual release of responsibility (GRR)

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Introduction

Systematic thinking is critical in everyday life. There are many problems we face, such as weather, pandemics, or economic problems. These problems must be solved using systematic thinking (Eidin et al., 2020). In addition, Epidemics COVID-19 can be dealt with using systematic thinking. Systemic thinking must first look at the big picture. Then look at the structure of how the system works. And when you want to solve a problem, look for the "tipping point" or "leverage point" to be found. (Worasom, 2020) From what the doctor said, the researchers realized the importance of systematic thinking. If a person can develop this skill, it can help to solve big problems in the world in the future. Moreover, if mentioned in mathematics, systematic thinking is also very necessary. This is because concepts in mathematics are systematically connected (Reda, 2021). Systematic thinking can also help students understand complex real-world problems (Karaarslan & Teksöz, 2023). Additionally, systematic thinking is in line with the skills needed in the 21st century as well.

According to Konold (1995), reasoning in everyday life relies heavily on probabilistic predictions. Many professions, including financial risk analysts, actuaries, natural disaster assessors, medical professionals, and political analysts, utilize principles of probability to make essential decisions. For instance, financial risk analysts evaluate investment risks, actuaries calculate insurance premiums based on probabilistic models, and medical professionals interpret health risks using probability. Even in simple, everyday decisions, probability serves as a fundamental basis. For example, when deciding whether to dine at a restaurant, one might observe the number of customers and infer the restaurant's popularity and quality based on this probabilistic assessment. Moreover, probability is not only vital in daily life and various professions but also serves as a foundational concept in mathematics, particularly in the field of statistics. In statistics, probability forms the cornerstone, enabling researchers to draw conclusions and analyze data (Casella & Berger, 2021). Therefore, understanding probability is crucial for both practical decision-making and further learning in mathematics and the related disciplines.

However, it has been found that many university students still harbor significant misconceptions about probability lessons (Khazanov & Prado, 2010). These misunderstandings may stem from students' experiences in various contexts, such as at home or in games, or may arise from misinterpretations during classroom instruction from the basics in secondary school (Jones et al., 2007). Additionally, the research by Kaplar et al. (2021) indicates that a substantial number of students continue to misunderstand probability concepts, with only 35% of test taker's ability to explain their answers correctly. These researchers emphasize the necessity of fostering a deeper understanding of probability. According to Ang and Shahrill (2014), students' misconceptions and difficulties at the secondary school level in this subject may be linked to their cognitive processes, learning methodologies, or problem-solving steps. Guiding students through the steps involved in solving problems could potentially alleviate

these misunderstandings. Further research suggests employing teaching methods that incorporate visual aids, practical activities, and systematic problem-solving techniques. Breaking down problem-solving into smaller steps can help students develop a robust foundational knowledge (Leung and Baccaglini-Frank, 2017). Moreover, the mathematics textbooks for Mathayom Suksa 3 provided by the Institute for the Promotion of Teaching Science and Technology of Thailand, in their lessons on probability, emphasize the importance of deriving results from random experiments. This systematic approach to counting is crucial; failing to use systematic counting methods may lead to incomplete or incorrect results. (IPST, 2563)

There is a teaching technique known as the Concrete-Pictorial-Abstract (CPA) strategy. The CPA strategy is an effective mathematics teaching method consisting of three steps. The first step involves learning through real objects, referred to as “Concrete”. The next step involves learning through visual representations, known as “Pictorial”. The final step involves learning through abstract concepts, referred to as “Abstract” (Mahayukti et al., 2019). Previous research has demonstrated that this approach helps reduce students' misunderstandings of mathematical concepts (Salimi et al., 2020) and assists in building a solid foundation for more advanced lessons (Azmidar, Darhim, and Dahlan, 2017). Additionally, a study by Sarfo et al. (2014) found that the CPA approach leads to more effective academic outcomes. Consistently using this teaching format helps students become familiar with the concepts and enhances high-level thinking skills (Robert, 2020). The Concrete-Pictorial-Abstract approach systematically builds students' understanding from tangible experiences to abstract reasoning by fostering foundational understanding, visual and spatial reasoning, logical reasoning, problem decomposition, reflective thinking, and the transfer of knowledge, the CPA approach significantly enhances systematic thinking. This progression helps students develop a structured and logical approach to problem-solving, enabling them to tackle complex mathematical concepts with confidence and competence. However, a study by Riccomini (2010) noted that transforming visual representations into abstract symbols can be challenging without teacher guidance. Therefore, teachers should have appropriate strategies to help students understand abstract concepts more easily. Furthermore, research by Salimi et al. (2020) emphasizes the importance of teachers paying special attention to the stages of the CPA approach, particularly in transitioning from the Pictorial to the abstract.

Among the obstacles encountered in teaching using CPA techniques, previous research has identified that many students struggle to make connections when transitioning from pictorial representations of abstract concepts. Students often find it difficult to write mathematical symbols without teacher guidance (Riccomini, 2010). To address this issue, the Gradual Release of Responsibility (GRR) technique has been introduced to promote more effective teaching.

The GRR teaching method gradually shifts responsibility from teachers to students through four

distinct phases. The first phase, "I do it", involves the teacher taking full responsibility, acting as a guide, and providing examples or demonstrations. The second phase, "We do it", sees students taking on more responsibility while still being supported by the teacher, working together on tasks and experiments. The third phase, "You do it together", emphasizes collaborative learning, where students exchange knowledge and assist each other, thereby enhancing their collaborative skills. The final phase, "You do it alone", requires students to apply their knowledge independently without assistance (Panlaan, 2019). By gradually shifting the responsibility of learning from the teacher to the student, the GRR approach nurtures students' ability to think systematically, enhancing their problem-solving skills, and critical thinking.

Of all the problems mentioned above, problems in the lesson on probability and the current world situation require accelerating the development of systematic thinking. To enhance analytical thinking and systematic problem-solving abilities, previous research suggests that students' understanding of probability lessons should be developed through more concrete teaching methods and a step-by-step instructional approach. According to what mentioned above about CPA and GRR techniques, both were proved as the effective teaching methods with different aspects on their benefits and limitations. Although a review of the literature reveals no existing research that integrates the CPA approach with GRR, the researcher proposed that combining these two techniques can address this gap. Therefore, this study aims to enhance CPA by combining it with GRR to overcome their limitations and developing more effective teaching techniques, which potentially can foster students' systematic thinking in learning probability concept in sustainable ways.

Objectives

The aims of this study were to (1) compare students' systematic thinking in lessons on probability using Concrete-Picture-Abstract (CPA) teaching strategies combined with Gradual Release of Responsibility (GRR), and (2) compare the development of four aspects of systematic thinking of students using Concrete-Picture-Abstract teaching strategies (CPA) combined with Gradual Release Responsibility (GRR).

Research hypothesis

1. Students improve their systematic thinking and the average score of students' systematic thinking after using the approach is higher than before using the approach.
2. Students improve their 4 sub-skills of systematic thinking and the average score of the students' 4 sub-skills of systematic thinking after using the approach is higher than before using the approach.

Research Framework

Working process of CPA combined with GRR

Concrete-Picture-Abstract (CPA) is a systematic learning approach. It's a gradual process. It can be composed of 3 steps. The first step is learning through real objects called concrete. The second step is Learning through pictures is called pictorial, and the final step is learning in an abstract form or mathematical symbols, called abstracts (Mahayukti et al., 2019). From the literature review of previous research, the researcher found an obstacle in the CPA teaching approach, namely that students have difficulty linking images into abstracts (Riccomini, 2010). The researcher therefore brought in the GRR teaching method to help promote it during the picture and abstract stages of CPA because the GRR teaching method gradually gives students more responsibility (Panlaan, 2019). It is divided into 4 phases. The first phase is that I do it. In this phase, the teacher will directly lead the way for the students. This phase can help students understand more about how to do things. The next phase is We do it, which means the teacher and students think together. In this phase, students will think along with the teacher. Students will try to think on their own but still have the teacher to guide them. The third phase is that you do it together, meaning that students learn together in groups and express their opinions while teachers have a lesser role in this phase. Students can think more on their own. And in the last phase, you do it alone, every student will try to think for themselves. I didn't get any help from the teacher. (Panlaan, 2019; Whittaker, 2016) The researcher therefore expects that the GRR teaching method can help students gradually understand the changes in CPA more smoothly than before. This is because the teachers lead in the beginning and gradually allow students to do it on their own in the end.

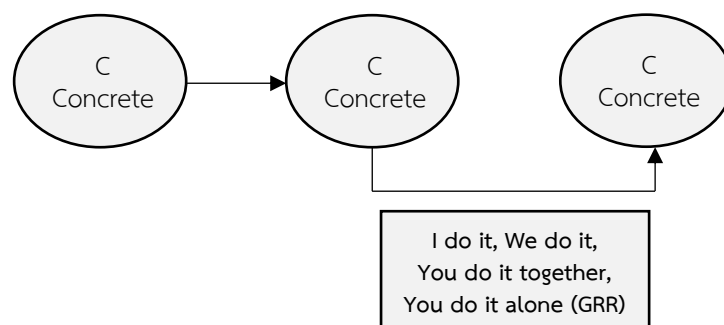


Figure 1 Working process of CPA combined with GRR

Systematic thinking is thinking step by step. It is a rational thought process. It is a problem-solving cycle that can organize actions and thoughts into a system. It solves problems by dividing the problem into parts and considering solving the problem part by part according

to the steps specified getting effective results. Systematic thinking is divided into four sub-skills (Al-Saeed & Al-Nimr, 2006):

1. Recognizing relationships skills: Students must be able to explain relationships among variables or relationships among given events in the lesson on probability.
2. System analysis skills: Students must be able to detect system errors or the point of error in finding probability. They must be able to identify where the error is, why it is wrong, and how to correct it.
3. System installation skills: Students must know several methods for solving problems and must be able to choose the appropriate method for finding the answer.
4. System evaluation skills: Students must be able to judge whether an answer is correct or not and explain how to check the answers.

Research Methodology

This research employed a pre-experimental design. The researcher initially tested the students' systematic thinking. Subsequently, students learned a lesson in Probability through the CPA Combined with GRR teaching strategy. Afterward, students were tested again to assess their systematic thinking post-learning using the CPA Combined GRR teaching strategy. All students were informed about the exam information and learning quotient, and all data collected were guaranteed to be kept confidential. The identity of the sample group was not specified.

1. Participants

This research employed the Concrete-Picture-Abstract (CPA) teaching strategies combined with Gradual Release of Responsibility (GRR) to develop systematic thinking among 40 Matthayom Suksa 3 (Grade 9) students at Suksanaree School, selected using cluster random sampling. The total population is 5 classrooms, with a total of 200 students. All students will be divided into 40 student classes with a variety of backgrounds and then randomly selected into 1 group of 40 students. The reason for focusing on this student level is that the probability lesson targeted for improvement is part of their curriculum of Matthayom Suksa 3. Furthermore, these students were encountering the subject of probability for the first time, making it essential to provide correct foundational learning to prevent future misunderstandings.

2. Research tools

The research instruments included: (1) a learning management plan consisting of 8 plans using the CPA technique combined with GRR in the context of probability, and (2) pre-test and post-test assessment exams.

2.1 Lesson plans

The content used in the research was a lesson on Probability within the mathematics subject of Matthayom Suksa 3. At the outset of creating the learning management plan, the focus was on ensuring the learning outcomes aligned with systematic thinking principles. The instructional

process was guided by CPA and GRR methodologies. Once the learning plans were developed, their quality was evaluated by sending all 8 plans to five experts for review. The evaluation involved assigning scores on a scale of 1 to 5. The average score of all 8 learning plans was between 4.5 - 4.7 and providing additional suggestions for improvement. These plans were refined based on the experts' feedback before being implemented in the research.

2.2 Written test of systematic thinking

The tests measure students' systematic thinking was created in two sets: a pre-test and a post-test. Both tests included parallel questions. Each test contained three main questions: Question 1 comprised four sub-questions, Question 2 had no sub-questions, and Question 3 included three sub-questions. All sets of questions were checked for Index of Item-Objective Congruence (IOC) quality by five experts, with scores of +1, 0, and -1. Any question with an IOC value lower than 0.5 was discarded. The results of the quality checks by the five experts found that all items had an IOC value of 1. Therefore, all questions passed the criteria and could be used. In order to assess the instrument's quality, the test was tried out with students who had studied before and calculated to check the quality of the instrument. All three test questions had reliability values between 0.85 and 0.92, difficulty values between 0.37 and 0.52, and discrimination values between 0.63 and 0.84, according to the inspection's findings. It was determined from all these findings that every test question met the requirements can be used in research. Consequently, the test items underwent thorough quality checks by experts and met all required criteria, ensuring their validity before being administered to research volunteers.

3. Data collection and Data Analysis

Data collection is done by having students take a pre-test and when learning has been completed students will be required to take a post-test. The test will be subjective. To see troubleshooting steps and analyze student scores by dependent t-test.

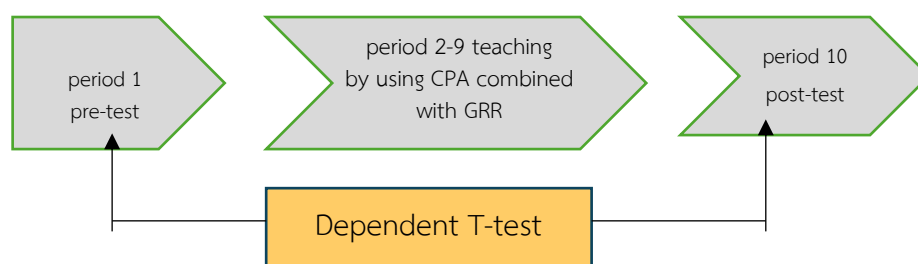


Figure 2 Summary diagram of the study

The data collection process in this research began by administering a pre-test to measure students' systematic thinking. Following this, students underwent instruction using the CPA technique combined with the GRR teaching approach in the lesson on probability. At

the conclusion of the learning process, a post-test was administered to assess the development of systematic thinking after the instructional intervention. The exam was evaluated using a rubric, and the pre-test and post-test scores were compared using a dependent t-test to analyze the statistical significance of the results.

This research strictly adheres to student privacy rights and confidentiality. All personal information is treated with utmost confidentiality, and no student personal data will be disclosed. The researcher has undergone training in human research ethics, and this research has been approved by the Institutional Review Board (IRB) prior to data collection involving students.

Research Results

1. Results of students' development systematic thinking in probability through CPA techniques combined with GRR

Comparability of data was checked by means of averages and standard deviation of systematic thinking scores before and after studying, and the data were compared using a dependent t-test. The results of data analysis are as follows.

Table 1 Difference of mean scores and standard deviations of systematic thinking between the Pre-test and Post-test

Group of Experiment	Full Score	$\bar{x}$	S.D.	t	p
Pre-test	22	3.43	6.61	25.27**	.00
Post-test	22	18.51	13.63		

** $p < .01$

From table 1, the t -value is 25.27 with a P -value (1-tailed) of .00, which is less than .01. Therefore, the null hypothesis (H_0) is rejected, and the alternative hypothesis (H_1) is accepted. This result supports the hypothesis that students' systematic thinking improved after using the CPA learning technique combined with the GRR approach in probability lessons.

2. Results of students' development of all 4 sub-skills of systematic thinking in probability through CPA techniques combined with GRR.

The comparability of data was assessed using the averages and standard deviations of systematic thinking scores in each of the four sub-skills, both before and after the study.

The data were then compared using a dependent t-test. The results of the data analysis are as follows.

Table 2 Difference of mean scores and standard deviations of the 4 sub-skills of systematic thinking

Skills	Group of Experiment	Full Score	$\bar{x}$	S.D.	t	p
Relationships	Pre-test	6	2.68	2.02	6.36**	.00
	Post-test	6	4.25	1.01		
Analysis	Pre-test	9	0.60	2.50	18.76**	.00
	Post-test	9	8.05	4.66		
Installation	Pre-test	5	0.13	0.32	23.20**	.00
	Post-test	5	4.66	1.30		
Evaluation	Pre-test	2	0.03	0.03	15.07**	.00
	Post-test	2	1.55	0.41		

** $p < .01$

From table 2, the P -value (1-tailed) is .00, which is less than .01. As a result, the null hypothesis (H_0) is rejected, and the alternative hypothesis (H_1) is accepted. Therefore, it is consistent with the hypothesis that students' 4 sub skills of systematic thinking improved after using CPA learning combined with GRR in probability lessons.

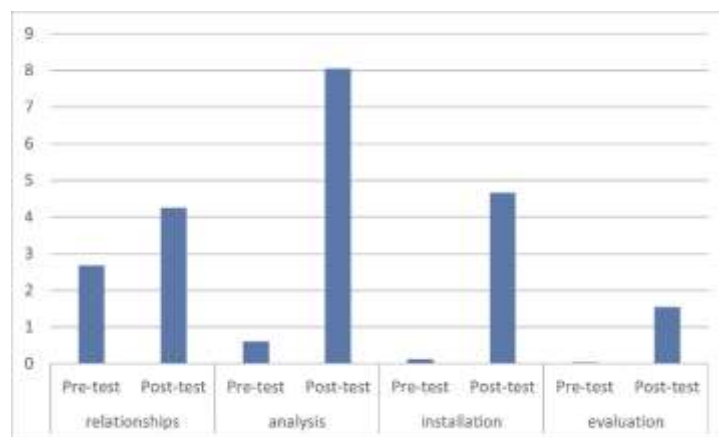


Figure 3 Difference between Pre-Test and Post-test scores of the 4 sub-skills of systematic thinking.

Conclusion

Concrete-Picture-Abstract (CPA) teaching strategies combined with Gradual Release of Responsibility (GRR) can effectively develop students' systematic thinking in probability. The

scores of systematic thinking after instruction using CPA combined with GRR were significantly higher than the pre-instruction scores at the .01 level. Students can write and present their ideas in a step-by-step manner, demonstrating an improved systematic thinking process compared to before learning with the CPA process combined with GRR.

Concrete-Picture-Abstract (CPA) teaching strategies combined with Gradual Release of Responsibility (GRR) can help develop the four sub-skills of systematic thinking in probabilities. Scores of the four sub-skills (i.e., recognizing relationships, system analysis, system installation, and system evaluation) were significantly higher after instruction using CPA combined with GRR compared to the pre-instruction scores at the .01 level. When studying the full cycle of the CPA teaching approach combined with GRR, it was observed that this teaching method effectively developed all sub-skills of systematic thinking in most students.

Discussion

The analysis of quantitative results revealed that students' systematic thinking scores improved significantly, Post-test scores markedly higher than Pre-test scores. This indicates that the combination of CPA and GRR teaching techniques effectively enhances students' systematic thinking abilities. These findings align with Saifur et al. (2014), who reported that the CPA approach improves student academic performance. Additionally, the results support Robert (2020), who found that CPA teaching methods are effective in enhancing students' thinking skills. Furthermore, the findings are consistent with Panlaan (2019), who observed significant increases in student academic performance following the implementation of the GRR teaching method.

According to research by Azmidar, Darhim, and Dahlan (2017) and Read, Landon-Hays, and Martin-Rivas (2014), CPA and GRR teaching methods can establish a strong foundational understanding for students, enabling them to progress to more advanced levels of learning. This research supports those findings, as it shows that initially, students were confused about the content. However, through the CPA and GRR methodologies, they were able to grasp the fundamental principles of probability more effectively. For instance, students who struggled with problems involving random selections, such as picking 2 balls from a total of 3, initially had difficulty starting the problem-solving process. Yet, with practice using CPA and GRR, students were able to accurately solve these problems regardless of changes in the scenario. The teaching process of this research does not only make students understand probability in terms of content alone, but also develop better systematic thinking as the students must systematically think about their answers to write in the test. The generated questions require students to think step by step and analyze events, not just understanding and answering questions alone. Therefore, writing expresses ideas and express

their step-by-step thinking. The researcher hypothesizes that the critical factor in creating a basic understanding is that learning through the Concrete approach provides students with direct experience from various activities. Additionally, using the GRR allows students to learn gradually alongside the teacher. This method helps students continuously absorb systematic, step-by-step concepts, enabling them to gradually become more independent. Ultimately, students will develop the ability to think systematically and follow steps automatically. This demonstrates that CPA and GRR not only solidify the foundational knowledge but also enable students to adapt and apply their understanding to varied situations. Thus, this research confirms that CPA and GRR teaching methods effectively build a solid knowledge base, facilitating students' progression to higher levels of learning. It also helps develop better systematic thinking.

From the issues identified with CPA teaching guidelines, Putri, Misnarti and Saptini (2018) found that the CPA approach struggled with linking images to abstract concepts. To address this limitation and enhance the CPA method, this research integrated the GRR approach. The combination proved effective in improving students' understanding and strengthening their ability to think systematically and follow the steps. Initially, students may have difficulty understanding the questions or knowing the appropriate thought process. However, after progressing through the first two stages of GRR, they develop a solid knowledge base that facilitates further self-problem solving. Some students demonstrated a robust foundation, enabling them to tackle diverse scenarios; However, this was not universally achieved. Certain student groups continued to face challenges in understanding the material, indicating a persistent minor issue within the classroom setting.

In terms of classroom behavior, the research found that most students were actively engaged and enthusiastic about the lessons. The classroom environment was positive, with students showing excitement about the activities planned for each day. The incorporation of challenges and interactive activities fostered an atmosphere that encouraged participation and facilitated a deeper understanding of complex concepts. This aligns with the findings of Putri, Misnarti, and Saptini (2018), who reported that students receiving CPA instruction demonstrated higher levels of enthusiasm compared to those who experienced traditional learning methods. Additionally, the results are consistent with Saligumba and Tan (2018), who found that the GRR teaching approach significantly improved student recognition and effectiveness in learning. Observations from this study confirm that the combined CPA and GRR methods not only stimulate student interest but also enhance their ability to grasp difficult mathematical concepts more readily than traditional methods. This supports Watt's (2013) research, which indicated that such teaching methods can alleviate the challenges associated with understanding mathematics.

This research addressed the limitations of the CPA approach identified in previous studies,

specifically the challenges associated with transitioning from pictorial to abstract thinking. By integrating the GRR method, these issues were largely mitigated, leading to improved conceptual understanding of probability among students. The combined approach allowed students to apply the same concepts to various situations and successfully navigate the transition from pictorial to abstract representations. Quantitative results show that all students' scores improved after employing this teaching method. However, students' answers in the Post-test exhibited similar patterns, reminiscent of products from the same factory, suggesting a potential issue of conformity rather than independent thought. This raises concerns about the possibility of students following predetermined paths too closely, which might hinder the development of original ideas. This is contrary to Read, Landon-Hays, and Martin-Rivas (2014), who suggested that GRR methods promote independent thinking. The results indicate that while GRR effectively enhances understanding, it may also inadvertently limit creative problem-solving by overly guiding students.

Next, the researcher will discuss the four sub-skills of systematic thinking, beginning with the relationship skills. In the Pre-test, it was observed that most students scored between 2 and 3 points out of 6 points. At this stage, students were able to explain relationships among various elements using simple language in their written responses. Notably, 5 students were unable to provide any answers for this section. After completing the study and taking the Post-test, the researcher found that most students' scores had improved to a range of 4 to 5 points. Students were now able to explain relationships more logically and effectively, applying the principles of probability with greater accuracy. Importantly, no students were unable to answer this section as they had been at the beginning.

The second skill is system analysis skills, which involves students analyzing situations, verifying the correctness of events, and making corrections when errors occur. In the Pre-test, only 8 students provided answers in this section, with scores ranging between 2 and 3 points out of a possible 9 points. Many students indicated that they were unable to perform this task. However, by the end of the study, most students were able to achieve full marks. They demonstrated the ability to identify errors, explain why they were incorrect, and make necessary corrections. Despite this improvement, a few students still struggled to achieve full scores. Some expressed that although they could identify mistakes, they were unsure how to correct them. This highlights a need for more targeted instruction for these students.

The third skill is system installation skills, which involves students demonstrating their problem-solving methods and summarizing their answers, like general math problems. In the Pre-test, only 2 students were able to show their thinking processes, while many students reported that they did not know how to approach or think through the problems. However, after instruction using

CPA and GRR teaching methods, the Post-test results revealed that 35 students achieved full marks. Students indicated that this section was the easiest part of the exam. They were able to effectively articulate their thought processes and summarize their answers correctly. This suggests that students developed a strong foundational understanding of probability concepts. Once the fundamental concepts are grasped, students can apply their knowledge to solve problems in various contexts.

The last skill is system evaluation skills, which involves the ability to verify whether the answers provided are correct. In the Pre-test, only one student was able to explain how to perform this preliminary check. Most students were divided into two groups: those who believed there was no method for verification and those who thought a method existed but were unsure how to apply it. After learning through the CPA and GRR teaching methods, many students were able to address this section successfully.

The overall analysis of the four sub-skills demonstrates that after instruction using CPA combined with GRR, students showed a marked improvement in their mathematical understanding. This aligns with findings by Watt (2013) that effective teaching methods can enhance students' mathematical thinking, and supports research by Azmidar, Darhim, and Dahlan (2017), and Robert (2020) which indicates that such approaches build a solid foundation and significantly improve students' thinking skills. Moreover, students not only develop their understanding of the subject but also enhance their systematic thinking. This improvement is evident from their written responses to questions, which demonstrate a clearer, step-by-step approach to problem-solving. Following instruction using the CPA and GRR methods, students were able to apply a more systematic approach to counting, which led to more accurate calculations of probability. This progression highlights their ability to think systematically and articulate their thought processes effectively.

Recommendations

1. Some suggestions for teachers to apply the results of the research

Given the positive outcomes of this research, the teaching approach employed has demonstrated effectiveness in enhancing students' systematic thinking in probability lessons. Therefore, the researcher recommends that Teachers incorporate this teaching method into their own classrooms. While the specific content of the lesson may vary, teachers are encouraged to adapt the instructional steps to fit their subject matter. This adaptation will support improved learning outcomes and foster better systematic thinking among students.

2. Some suggestions for further research

The overall results of the research are positive, but the researcher encountered certain weaknesses with the GRR approach, specifically that it may impose too rigid a framework on

students. Future research should consider developing improved teaching techniques, possibly by integrating new methods with CPA or exploring alternative approaches to complement GRR. This could help foster greater creativity and independent thinking among students. Additionally, it may be beneficial to design communication strategies for each step of GRR that avoid making the knowledge too patterned, thereby enhancing the effectiveness of the teaching method. Therefore, for future research, it may be possible to explore ways to integrate CPA with other teaching techniques to further develop student learning.

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