

# Abundance of Formative Assessment

## A Local, Traditional Lesson Mode In Elementary Mathematics

### With An Abundance of Formative Assessment

Minato, Saburo<sup>1\*</sup> and Tanaka, Seiyuu<sup>2</sup>

<sup>1</sup>Emeritus, Akita University, JAPAN

<sup>2</sup>Faculty of Education and Human Studies, Akita University, JAPAN

#### Abstract

In this article, we introduce a local, traditional lesson mode of elementary school mathematics, developed round 1965 in Akita, Japan, and from beginning, it has had an abundance of formative assessment. For the sake of explanation of this fact, a realistic lesson, which is named Sheet Learning, is reconstructed depending on our memories. Main findings of this article are that installation of self trial of solving problem in the process of this lesson pushed the process to use formative assessment, the teacher worked as in a lesson study in this teaching process, and that teachers in Sheet Learning must be professional.

**Keywords:** Activity Oriented, Akita, Formative Assessment, Japan, Lesson Study, Problem Solving, Sheet Learning, Professional.

#### Introduction

##### 1.1 Formative assessment and Sheet Learning of Akita

The aim of this article is, introducing so called Sheet Learning enduring in Akita, Japan, we argue that this learning process in elementary school mathematics has had an abundance of formative assessment since the beginning of it, and installation of student's self trial of problem solving activity pushed the process to use formative assessment, and discuss some required qualities of teachers in Sheet Learning. Here, formative assessment is "frequent, interactive assessment of student progress and understanding to identify learners needs and adjust teaching appropriately(OECD, 2005, p.21)." Whereas well known summative assessment is most visible assessment used to measure student's achievement, mainly for proof of completion of learning, certification and accountability, dealt with at the end of a unit, a term, or a school year, formative assessment is dealt with in the fly of learning process in the classroom for modifying or promoting student's learning. In England, "in the 1970s and 1980s, a number of research projects had explored the way in which assessment might support learning"(OECD, 2005, p.129). so formative assessment was not new to the British national agenda when the Enculturation Reform Act (ERA)was implemented in 1988.

Now formative assessment is a topic of much concern for expecting to improve worldwide education. Focusing on classroom practise greater degree than is usual in OECD studies, OECD(CERI, 2005, p.13) study "looks at the practise of formative assessment in classrooms and schools in eight education systems."

\*Corresponding author. Tel: + 81-18-832-6448  
E-mail address: s-tanaka@gipc.akita-u.ac.jp



Although the results of case study schools in formative assessment of these systems are reported in 'Formative Assessment' (OECD, 2005), there is no one from Japan, as Janet Looney of OECD(CERI) regrettably writes in Japanese version of the book published in 2008.

Because of the following two facets, Akita Prefecture (in the sequel, simply refer as Akita), which is in the northeastern region of the mainland of Japan and has population of a million, is noted for the elementary school mathematics teaching in this country. One is to perform the highest achievement of mathematics within the National Assessment with the beginning of the 2007 school year. With the lack of administration of the 2011 school year, owing to the Great East Japan Earthquake and Tsunami in this year, Akita has successively performed the highest average scores on primary mathematics in both of the Tests: Primary Math A (basic knowledge and skill) and Primary Math B (mathematical literacy), from the beginning of the 2007 to the 2015 school year (when we are writing this article).

The other is that there has been a characteristic teaching-learning mode of mathematics, developed by mathematics teachers and supervisors of Akita, first started in 1962 and is called Sheet Learning, which name is originated from the use of so called Learning Sheet, a kind of worksheet with definite style, of the size B4, or A3, made by every teacher and distributed to every student in the lesson. The half century of our teaching and learning experience after the development of Sheet Learning may be divided into two periods, the former half (1962-1989) of pursuing better lesson of Sheet Learning approach within its framework, and the latter half (1990-2015) of some trials of improvement of Sheet Learning affected by outside educational requirements. For the aim of this article, we deal mainly with the former half period of Sheet Learning.

For the former half period, the formulation of Sheet Learning was set up, and groups of several teachers experimented with it in Akita. Then Sheet Learning approach was implemented owing to information through private networks, and study on multilevel (small or large, narrow or wide, informal or formal) workshops for teachers most of all with demonstration of lesson. Its astonishing rapidity of implementation was essentially based on teachers' wanting their lessons to improve. Anbo (1968) theorized and described in thorough manner the foundation of Sheet Learning, the way of designing Learning Sheet, its teaching process, and the effects, under a Japanese longing for caring and systemic nature for education, with making reference to the philosophy of J. Perry (England: Mathematics Education to All), G. Kerschensteiner (Germany: *Arbeitsschule*), J. Dewey (America: *Learning by Doing*), and J.S.Bruner (America: *Learn How to Learn*). He stressed the need to observe and record for each students in the classroom, like a clinical recording of each patient under diagnosis and caring treatment of a doctor. For this, he insisted upon the use of now called formative assessment.

Sheet Learning is basically not intended to introduce a special curriculum, but a kind of teaching-learning methods, and its lesson is characterized as follows:

F1: The lesson is based on the national curriculum, and the contents almost depend on the textbook, with small modification, if it makes the lesson more suitable for the learners. This small modification is ordinarily assumed to be permitted in this country. In fact, there is a general belief that the person who is capable of knowing students best is the teacher,

F2: After a bit of revision on the previous lesson or sharing the aim, the lesson begins with students' activity, not with teacher's lecture,

F3: This students' activity is of self trial of solving problem, and it is important and avoidable,

F4: Discussion in the lesson is necessary for deepening and developing concepts and ideas,

F5: Learning Sheet with a definite style is designed by the teacher, and used in the lesson (An example of the lesson and Learning Sheet will be shown in chapter 2 of the article).

Although the aim of this article is concerned with Sheet Learning for the former half period, it is not useless to describe some transition from the beginning. In fifty years from the beginning of Sheet Learning, the lesson has been gradually altered in the direction of the emphasis of for all, from the emphasis of discovery and creativity, and in the latter half period, some improvements, or ingenuity have been made. Some years ago, substantially small in education, but a large and decisive change in the use of Sheet Learning was taken by making students use their notebooks, substituted for Learning Sheets. Then Learning Sheet has almost disappeared from lessons, leaving the above mentioned characteristics except F5. The word Sheet Learning without Learning Sheet is contradictory, likewise the phrase 'Picturebook without a Picture'. We call the lesson Akita Mode of (Mathematical)Lesson, or sometimes Extended Sheet Learning, which inherits the above characteristics of F1, …, F4.

Recently, a researcher Yamamoto(2015) found some formative assessments in Akita Mode of Mathematical Lessons practised in two elementary schools in Akita under the research design of participated observation.

### 1.2 Framework of Sheet Learning

The framework of Sheet Learning consists of four Phases A, B, C and D, and are characterized as follows (with typical time period, suggested by Anbo, 1968):

- A: A bit of revision of preceding lesson, or sharing the aim, (5 minutes)
- Next Phase B is the main part of student's activity, which can be divided into B1 and B2.
- B 1: Self trial of solving problem, sometimes group activity is added, (10 to 15 minutes)
- B 2: Presentation of students' trial solutions with explanation and discussion for exploring more effective conclusion and deepening understanding, (10 to 15 minutes)
- C: Practicing some problems, applying the conclusion in B2, (10 minutes)
- D: Confirmation. (5 minutes)

An example of Learning Sheet, which is referred in chapter 2, is shown in Figure 1. Another example in the latter half period is in Minato (2015, P.18).

Sheet Learning belongs to Problem Solving and Discussion Mode, which has been developed after Lecture Mode (Master-Student relationships), and Questioning and Answering Mode (Minato, 1997, Minato and Yatsuyanagi, 2014). This Questioning and Answering mode is described by Stigler and Hiebert(2009, p.29) as German lesson(Developing Advanced Procedure).

### 1.3 Miscellaneous information about schooling of Japan

At the end of the introduction, we had better offer the readers some miscellaneous but useful pieces of information about schooling and others of this country. There is nine years' compulsory education from 6<sup>+</sup> to 15<sup>+</sup>, which can be divided into six years of elementary school and three years of junior high school. The National Course of Study that announces subjects, aims, hours, items, and fundamental teaching-learning methods, has been revised traditionally every ten years by the Ministry of Education, Culture, Sports, Science and Technology (MEXT). The textbooks are published by private publishers, but they must be approved by the Ministry. The hours (one unit hour is 45 minutes) of elementary mathematics are, 1st: 114, 2nd: 155, 3rd: 150, 4th: 150, 5th: 150, 6th: 150, and totally 869. Nowadays, the number of elementary school students in a class is not more than 35. In principle, every teacher of elementary school has his/her classroom, and is



responsible for teaching all the subjects, but sometimes he(she) leaves other teachers some subjects, mostly music, drawing and manual art, or physical education in exchange.

Although the description of this article depends on some extent on the textbooks published by Kyouiku Shuppan Publ.(Tokyo), they are written in our native language, and have not been translated into English. We would like to announce that there is a set of English version of elementary mathematics textbooks published by Keirinkan Publ.(Tokyo), available everywhere. These books published by two publishers are approved by the Ministry, and there are not decisive differences between these.

Strictly speaking, we seldom use corresponding Japanese to 'teaching' in English. We use 'Gakushuu Sidou' in Japanese, which means guidance of students' learning. But in this article we dare use 'teaching' for denoting teacher's behaviour or activity in the lesson. A similar case is emerged at the use of 'peer tutoring' in English. The corresponding words in Japanese to it is 'Manabi Ai' which means learning together.

## 2. Portraying a Sheet Learning Lesson and Teacher's Work

### 2.1 Preliminary notes

In the chapter, we would portray a Sheet Learning on the material 'Finding the Area of Trapezium' and formative assessment in it. Although following realistic lesson is a reconstruction of a past lesson in the former half period from our memories, it was not a special but ordinal lesson practised in Akita at that time.

First of all, we have to write some preliminary affairs on the assumption of what materials students have already learned in relation to this material, and how the teacher should work for preparation for the lesson. The assumptions are enumerated, as follows: Unit of area,  $1 \text{ cm}^2$ , Geometric figures of square, rectangle, parallelogram, and triangle, Ways of finding the areas of these geometric figures, Meaning of trapezium, the upper base, lower base, and the height.

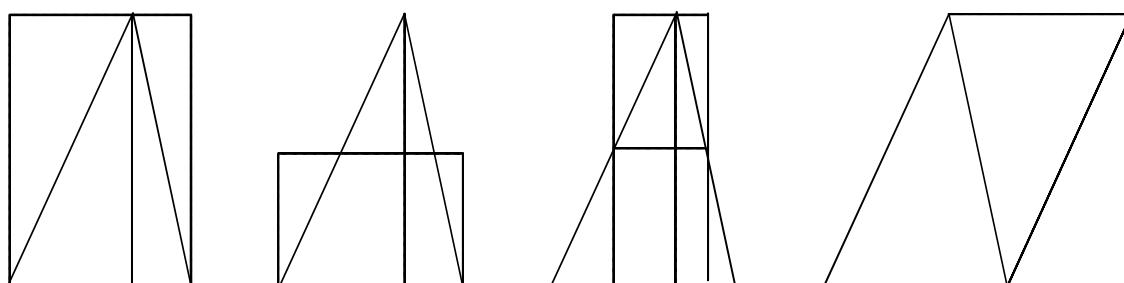
Preparation necessary for the teacher is not so simple in the case of Sheet Learning, and noted in the next section 2.2.

### 2.2 Preparation before the lesson

Before the lesson is taken, some preparations have been done by the teacher. In this article we describe these with explanations, and suggest that the preparatory works are similar to a part of so-called lesson study (Stigler and Hiebert, 2009, p.110).

#### 2.2.1 Reflection and decision making of an aim

Reflection on the preceding lessons on geometric figures, especially the last lesson, is done. In the last lesson, students learned the area of triangle, and succeeded it. They found the area of the triangle, using some ideas:



These are fundamental ideas of making a rectangle, two rectangles or a parallelogram, and splitting or moving, which will be available in a next lesson. Then the teacher decides the aim of the next lesson: 'Finding the Area of Trapezium' referring to the textbook of Elementary Math 5 published by Kyouiku Suppan Publ. (Tokyo).

### 2.2.2 Designing Learning Sheet

The teacher is going to design a Learning Sheet (a definite kind of work sheet, as shown in Figure 1) for the next lesson. As there are four Phases of students' activity (see 1.2), the teacher picks up suitable problems corresponding to expected students' activities in each Phase. This time, Phase A is for revision of finding the area of a triangle of the base 5cm and the height 4cm.

In Phase B, the teacher gives a problem, which students find ideas of the area of trapezium and discussion. Therefore the teacher has to select problems deliberately, making reference with the textbooks and others.

Considering students' various ideas of finding the area and well obtaining a general formula, the teacher decides that (upper base + lower base) is an odd number, and the height is an even number, and does not give any lengths of segments intercepted by perpendicular line to the lower base through the end point of the upper base. The problem given in Phase B: Finding the Area of Trapezium with the upper base: 3cm, lower base 8cm, and height 4cm. This trapezium is slightly different from that of textbook. The teacher designs Learning Sheet of the lesson (Figure 1), after deciding four problems for Phase C, and two problems for Phase D, referring to the textbook, and then prints it out.

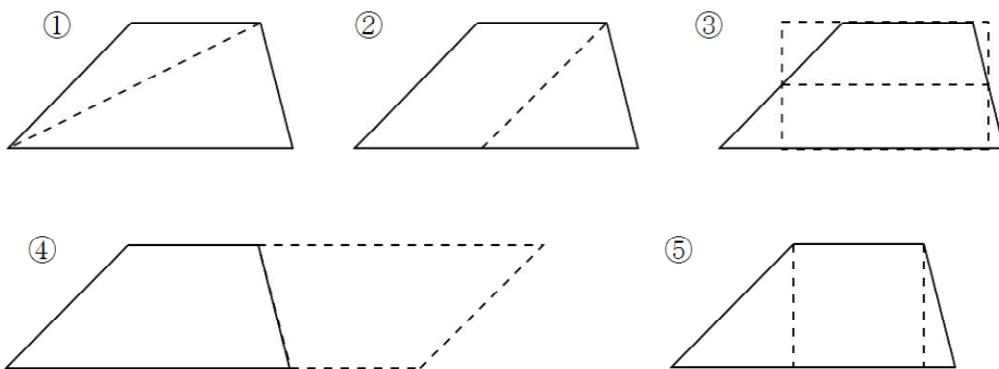
(Insert Figure 1, here.)

### 2.2.3 Previewing with teacher's anticipation

On the Learning Sheet, the problem of Phase A may be easily answered by all students.

At the beginning of Phase B1, confirmation of students' understanding of the meaning of trapezium, the upper base, lower base and the height is necessary. In Phase B1, following ideas are expected on the problem:

- ①: splitting it into two triangles,
- ②: splitting it into a triangle and a parallelogram,
- ③: splitting it into two trapezia with a centre line in the middle of the trapezium, parallel to the base, and obtaining a rectangle,
- ④: using same trapezia, making a parallelogram, and
- ⑤: splitting it into two triangles and a rectangle.





The way of ⑤ is not impossible, but difficult, so the teacher gives some hints and leaves students' future task, and does not refer to in the classroom. If anyone succeeds using ⑤, this way is referred in the next lesson.)

#### 2.2.4 Students' matter in hand

A memorandum of students' matter the teacher grasped is as follows:

Stronger performers: G, H, J,

Weakers: K, L, M,

Students less inclined to contribute: N, O, P, Q,

Need to care, speak to, and encourage: R, S.

Others: T, U, ⋯.

Some pieces of information on the above matter, gathered using formative assessment with anticipation, sympathy and deep understanding of students, are not hard data, but entirely tacit. The students' matter, as well as expected students' ideas of finding the area are noted with the symbols ①, ②, ⋯ on a handy size (B5 or A4) paper with chart of student seats. This sheet of paper is called Chart of Students' Seats (Zaseki Hiyou, in Japanese), which the teacher always bears during the lesson.

### 2.3 Realistic lesson of Sheet Learning

#### 2.3.1 Opening ceremony

Traditionally the lesson of Japan begins with a opening ceremony such that all students stand up, bow to the teacher, following the signal from the student monitor, then the teacher bows in return, as Stigler and Hiebert (2009, p.16) aptly described. Although the description makes easily image the lesson being a master-student relationship, there is not a lecture, but rich students' activity in the case of Sheet Learning.

#### 2.3.2 Phase A

Soon after the end of the ceremony, the teacher distributes Learning Sheet printed out yesterday to every student, and says, "Yesterday, we made the formula of finding area of a triangle. Could you find the area of the triangle on your sheet?" The students start working on the problem, and the teacher walks around quickly, without the lack of watching the sheets of students K, L, M, R and S. After walking around, the teacher says, "You are all well, and the area is ?" Students say  $10\text{cm}^2$  in unison. The teacher says, "Well, we are going to today's task in B."

-5 minutes passed (from the beginning of the lesson)

#### 2.3.3 Phase B1

The teacher draws a sketch of a trapezium on the black board, and pointing at the upper base and lower base, and says, "You know trapezium, don't you? What are these?" Student say, upper base, lower base, respectively. The teacher call student R, "Come to the black board, and draw the height of this." He draws it on the sketch using the symbols for a right angle. The teacher asks other students whether his drawing is suitable or not, and then says, "Yes, very good."

Under the direction of the teacher, the students are going to work on finding the area of trapezium in B. Bearing Chart of Students' Seats on a paper holder, the teacher walks around slowly this time, gazing used ideas of each student, ticking them with symbols ①, ⋯, ⑤ on the Chart, speaking to students R, S, and H, scaffolding (give hints) and encouraging students K and L. Whereas the teacher confirms that all students find the area at least one way, the teacher is not able to find the idea ④ on students' sheets.

The teacher says, "You are all well, but could you find another way for finding the area? Yesterday, you found the area of a triangle by the idea of adjoining two triangles. Could you apply the idea in this case?" After this scaffolding, student T hails. The teacher goes to the student, watch delightfully his sketch with idea ④. Some of the other students G, H, and U also hail silently, and they get this idea. There are some students having idea ⑤. and teacher scaffolds, but leaves them as their tasks. Then teacher nominates four students K (①), N (②), P (③) and T (④) to let them present their ideas.

- 18 minutes passed

### 2.3.4 Phase B2

They draw sketches and write their expressions for finding the area from the left to the right on the black board in the order of the teacher's nomination, explain their ideas and answer some questions. They are applauded by the teacher and the students.

The teacher ask student U to show and explain her idea, for in walking around the teacher has seen her sketch, which is to lengthen two sides of trapezium to intersect, obtaining two triangles. She tells, "I think my idea is also well, but I can not obtain the heights of the small and large triangles. The heights of these triangles must be decided, I think, but (Pause). Is there any one helping me?" There is no one responding to her request. The teacher says, "Her idea is splendid, but we can not find the heights now. When you learn 'similarities' of geometric figures in junior high schools, you can find the heights of these."

The teacher is going to characterize their ideas from the standpoint of geometric figure: using two triangles(K), a triangle and a parallelogram(N), a rectangle using a centre line(P), and two trapezia(T). Then the teacher says, "They are all well. We would like to classify the ideas by another way, and how about?" Student H replies that these ideas can be classify along the expression of numerals, or ways of calculation, and that the ideas of students N, P and T are only once in division, so they are simple. The teacher says, "Oh yes, we can classify following the way of division, especially with once or twice." The teacher is pointing at the geometric figures sketched by students N, P and T, and says, "Although the idea of N is very fine, it includes subtraction. The ideas of P and T are ways of using division once. We want clear geometric figure and simple calculation for obtaining a formula for the area of trapezium. Geometric figures of K and T is visibly simple, and the idea of T uses division once, so following the idea of T, we are going to obtain a formula for finding area of trapezium."

$$(\text{Area of Trapezium}) = (\text{Upper base} + \text{Lower base}) \times \text{height} \div 2$$

The students note this formula at the bottom of the space B on their Learning Sheets.

- 32 minutes passed

### 2.3.5 Phase C

Under the teacher's direction, the students start working in small groups, each of which consists of four students. One more direction given by the teacher is that the first problem in C can be solved by any idea you like, but for the rests students should be firmly followed by using the formula now obtained. Students show and watch together their ideas of finding the areas. They are visibly pleasant, exchanging ideas and opinions about goodness of them. There are much of peer tutoring, or learning together. The teacher quickly walks around groups affirmatively and delightfully, and speaks to students R, S and O. The teacher pats students K, P, T and U on their backs with a warm smile.

- 40 minutes passed

## 2.3.6 Phase D

Following teacher's direction, Students are released from grouping, and then work individually on two problems of Phase D. The teacher watches students K, L and M, confirming that they are well. Then the teacher speaks aloud to the students, "Well done." The teacher asks the answers, one is for student Q and the other for student M. Their answers are all right. The teacher says, "You may write your feeling or opinion on the lesson at the bottom of your Learning Sheet. After a short time has passed, the lesson ends with a closing ceremony, that is the students' expression of a feeling of gratitude for the teacher. The students push their chairs back, stand up, and bow in unison.

- 45 minutes passed

## 2.3.7 Other ways of the lesson

The above portrait of a case of Sheet Learning mode may be ordinal and standard, and there are some variations in the way of taking Sheet Learning. One of them is that Learning Sheet has some creative, difficult problems. Usually, such problems are installed in the space between C and D, then ordinarily named D (confirmation) is in these cases named E. Some able students tried to solve such problems after solving problems in C. Teachers who were strongly expected development of students' creative thinking were prone to deal with these problems. One of the lesson most different from the above example is to nominate students with making erroneous answers in Phase B1. The purpose of this harsh and terrible treatment is that the teacher strongly expected to share this erroneous but probable thinking, and to make reflection for deeper understanding of all the students. These lessons have been done sometimes in Akita. This treatment by teacher can only be done under the establishment of classroom culture such that students feel safe to make mistakes in the classroom. Certainly, "this is just part of the learning process" (OECD, 2005, p.170).

Teachers of Akita have thought that mistakes by students can make all the students gain more learning, and should be rich sources of learning. When the teacher walks around and finds a student's mistake or erroneous answer, the teacher nominate him(her), as an indispensable contributor and let him(her) write his(her) answer on the board. After the student's explanation of his (her) answer with a mistake, and some discussion, of course the teacher cares about the student, and tell him(her), "I greatly appreciate your presentation, and we can find and share an important and critical matter. Your presentation is very valuable for us all, so all of us should say thank you very much for your presentation and explanation."

In our country, there are some books referred to how teachers should encourage such students in the classrooms. One of the books (Hashimoto, et al. 2003) refers to whether answers are erroneous, incomplete or right, teacher should evaluate these answers in order of making much more learning. This book also refers that when there may be a student's murmur in the lesson, the teacher should tell him(her) that a clear expression is necessary. Moreover, teacher's picking up a valuable murmur requires teacher's wide perspective on the subject matters and mathematics. Bishop (1988, p.164) eloquently said, "..., because enculturation is an interpersonal process, the enculturator will be required to 'personify' the culture as much as possible," where enculturation and enculturator mean teaching and teacher (of mathematics) respectively.

In Japan, there have been some cases where university professors, supervisors, or prominent teachers are invited to meetings of lesson study, and practise model lessons. In these cases, if some kinds of Learning Sheet are used, or the lessons are practised in the mode of Sheet Learning, materials used are not restricted



within, but released from the curriculum and the textbooks. Such materials are called Throw-in Materials, and usually developed by themselves.

## Discussion

In this chapter, we would discuss the relation between formative assessment and Sheet Learning, teachers' work enlightened by the conception of lesson study, applicability of formative assessment to large classes, and quality of effective teachers of Sheet Learning.

### 3.1 Introducing formative assessment into Sheet Learning

In the above description of Sheet Learning, it is visibly clear that there are an abundance of formative assessment, and we would stress that the establishment of student activity and discussion is the mother of formative assessment in this lesson. An insightful book by OECD (2005, p.68) says, "Effective formative assessment approaches and techniques help to discipline and make transparent the teaching and learning process." Although we agree with this assertion as in case study schools, in the case of Sheet Learning we should tell the relation between formative assessment, and teaching-learning process in the reverse order. The installation of ten or more minutes period of students' self trial activity (Phase B1) made possible or forced to use formative assessment. Developers of Sheet Learning and the then teachers newly obtained this time period pursued effective usage of now called formative assessment.

In fact, at the beginning of Sheet Learning, there were some teachers who did not fully understand the value and usage of this period, and wandered at random in the classroom when their students were working in self trial activity. These teachers were advised by fellow teachers, the principal, or visiting supervisors, such that, "Don't stroll in the classroom, or you should walk around with clear aims at doing something important." There has been an educational technical term 'Kikan Shidou' in Japanese, corresponding to 'walking around,' 'wander around', or 'circulate around' (Stigler and Hiebert, 2009, p.39), which totally represents teachers' behaviour of stopping by watching student's Learning Sheet, aiming at observing his (her) students description of expressions, chart and numerals on it, scaffolding, and speaking to for encouragement with caring manner, gathering information for Phase B2, that is to say formative assessment.

Teachers of Akita believe that students are apt to engage in challenging problems in the lesson, if given problems are suitable for them, that is not so easy and not too difficult, and some other conditions, sharing aims, giving suitable time periods, being caring individuals and teacher's giving attentive manner to the classroom atmosphere, are preserved. Then the teacher obtains a period of Phase B1, being able to guide each student stopping by his/her seat, speak to, and encourage him/her. The teacher gathers many pieces of information not only for scaffolding in this period, but also for the next Phase B2 of orchestrating discussion. Moreover the teacher can know ways of students thinking styles, or childish thinking, and the students themselves. Every teacher believes that such knowledge will be valuable for his/her future life of a teacher.

Walking around (Kikan Shidou) seems traditionally to be very important and valuable in Japan, because of its three main functions: ① assessing for the lesson(scaffolding, pointing out erroneous thinking), and modifying of the lesson, ② sympathizing with students affirmatively(speaking to, encouraging, patting on students' backs or shoulders), and ③ teacher's learning of real states of students and their thoughts, which cultivates teachers for their professional development.

Stigler and Hiebert (2009, p.155) noted such assertion that thoughtful discussion is to be quite effective, “but unless one knows what expect from students, it is a scary way to teach, and success depends on making many split-second discussion about which student suggestion to follow up on and which to ignore.” One way to make discussion well orchestrate is to gather information each student’s understanding, who deeply understands and who not, who contributes. This is difficult for teachers to estimate and they have to gather such information using assessment.

Another way to dissolve this difficulty is to shift the key for effective teaching “from on-fly- and split second decision making to careful investigation and planning before the lesson” (Stigler and Hiebert, 2009, p.156). This teachers’ investigation and planning is described as ‘Preparation before the lesson’ in the section 2.2 of this article.

### 3.2 Teacher’s work enlightened by the conception of lesson study

The section of ‘Preparation before the lesson’ consists of reflection, decision making of the aim of the lesson, previewing with teacher’s anticipation and taking care of students. The teacher works similar to in a former part of so called lesson study, then the teacher practise classroom lesson, and will certainly reflect for the next after the lesson.

Stigler and Hiebert(2009, p.110)explained lesson study, as follows: “in lesson study (Jugyou Kenkyuu in Japanese), groups of teachers meet regularly over long period of time to work on the design, implementation, testing and improvement of one or several ‘research lessons’.” Although the preparation before the lesson described in section 2.2 is simple, extremely short in time, performed by only a teacher, and the teacher practises lesson without any outside observers, then reflects for the next lesson, the teacher’s intrinsic work does not differ from lesson study. We can call this teacher’s work Alone Situational Tiny Lesson Study, and this time, from a standpoint of view, the teacher may be a mini researcher of lesson as well as a teacher.

### 3.3 Applicability of Sheet Learning to large classes

It is interesting to note that Sheet Learning is applicable to large classes of 40, 50, or more students, which we have had considerable years ago. Contrary to presupposition that “it is not so easy to use formative assessment with large classes”(OECD, 2005, p.69) as especially secondary school teachers may protest, if formative assessment is used with activity oriented approach of teaching, teacher may teach without too difficulty (of course, not only elementary, but also junior high schools). Analyzing data from the TIMSS Video Study, Koon(2005, p.209) noted that the nature of activities talking place in the classroom is more important than the size of the class. This conclusion coincides with our past experiences of Sheet Learning and then using formative assessment, and it may be extended to classrooms with many cultural background.

### 3.4 Quality of effective teachers of Sheet Learning

Although humanity and gentle bearing on students are necessary for any teachers in any teaching mode, in the sequel, we would describe especially required quality of teachers taking Sheet Learning. Firstly, it is required for such teachers to have high abilities, especially in mathematics. Beeby(1980)performed an empirical study in developing countries, and concluded that less able teachers in knowledge were prone to do text-teaching (master-student relationship), and could not organize activity oriented learning and discussion. Anbo(1968, p.65), one of the important developers of Sheet Learning, wrote that generally speaking, Sheet Learning approach promises to be much effective processes of learning, but it is restricted such that the effectiveness of the lesson deeply depends upon teacher’s ability, especially in mathematics.



It was lucky for Sheet Learning and Akita that before Sheet Learning was developed, an educational regulation of this country had changed, and one of the requirements to be a teacher had been from the graduation of normal school to that of university. This legal transition in the certification with longer and richer study of mathematics of prospective teachers have ensured their sufficient abilities in mathematics.

Secondly, Sheet Learning is an activity oriented approach, so teacher is expected to have a suitable conception of mathematics. Dossey (1992) described that there are external (Platonic), and internal (Aristotrian) conceptions of mathematics, where external conceptions view mathematics as a pre-existing, external object, and internal conceptions view mathematics as a personally constructed or internal set of knowledge. He also described that teacher may have a great deal to do with the way in which mathematics is characterized in classroom teaching, and the internal conceptions are suitable for activity oriented or problem solving centred in learning. One of the author (Minato and Hamada, 1994) clarified that a teacher who has Platonic external) conceptions of mathematics is difficult to make subjective, activity oriented learning, and a teacher who has Aristotrian (internal) conceptions can well guide an activity oriented learning. Ernest (1998) almost agrees to this perspective.

Thirdly, teacher dealing with Sheet Learning must know the curriculum of all the grades, especially of elementary school in detail. Formative assessment in Sheet Learning makes the lesson suitable for students, and then the teacher has to change materials or items from those of the textbooks. This work is difficult for teachers and must be done in a complex system of education. Therefore teacher in Sheet Learning, and then using formative assessment must be able to review school curriculum.

A belief which has strongly and consistently followed from past Sheep Learning to today's Akita Mode of Mathematical Lesson, is that every teacher must be a mini writer of textbook. In addition to the assertion that every teacher is a mini researcher (section 3.2), teaching must be taken with a professionalism.

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