



A Study of Points of Teaching Proactive Use of Mathematical Representation in Arithmetic: Focuses on the Difference Between Arithmetic and Mathematics and Characteristics of Writing and Drawing

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Abstract

Focusing on writing, Shimizu (2012) found two difficulties contributing to students' inability to write mathematical representation. The author proposes ways to overcome these difficulties in the teaching of mathematics.

In this paper, the author also proposes ways to address these problems in the teaching of arithmetic.

As suggestions for ways to address these problems, the author cites the alleviation of rules and abstractions and the promotion of the enjoyment of writing.

The author proposes ways of guidance to encourage children to write positively.

(1) Children should be encouraged to write their own ideas, freely using various kinds of representation including Illustrative Representation, with the teacher gradually making specificity and directness a priority.

(2) Teachers should prepare contents which are interesting and understandable, thereby encouraging children's motivation to study autonomously by feeling the enjoyment of writing. The teacher should be aware of the need to carefully progress from the emotional in mind earliest stages of studying.

(3) Both teachers and children should be personally and positively engaged with the object of studying.

(4) Both teacher and children should be felt to value the quality and importance of writing.

(5) By the children seeking of higher goodness (value) of arithmetic, and children's higher enjoyment of studying, children should study to use new representation over time, and in as natural a way as possible.

In proposals (1) - (5), while the author emphasis is on the early stages of studying in arithmetic and mathematics, it was necessary to take into account the unique characteristics of arithmetic.

Keywords: Studying of Arithmetic, Mathematical Representation, Writing and Drawing, Proactive Use

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Introduction

It is clear that it is important for children to write when they study arithmetic and mathematics. The author wants to enable students to describe how they arrived at a certain solution. The author wants the students to use proactive mathematical representation, even if the teacher doesn't tell them to do so.

Focusing on writing, Shimizu (2012) identified two difficulties preventing students from writing which are caused by its characteristics. The author proposed ways to overcome these difficulties in the teaching of mathematics by using mathematical representation proactively.

The purpose of this paper is to consider ways to overcome difficulties of writing not only in the teaching of mathematics but also in teaching arithmetic. In this regard, the author proposes ways to teach children to proactively use mathematical representation when studying arithmetic. Specifically, the author considers differences between mathematics and arithmetic when teaching these subjects.

The Research method considers the study of arithmetic itself as well as mathematical representation in the study of arithmetic, with the basis being Shimizu (2012), and the study of arithmetic.

It should be noted that the word "writing" as used here refers to Linguistic Representation and Symbolic Representation, while the word "drawing" refers to various things including Illustrative Representation, Linguistic Representation, and Symbolic Representation.

Consideration by the Comparison of the Studying of Arithmetic and Studying of Mathematics

Hirabayashi (1993) describes the difference between arithmetic and mathematics in this way; the study of what is needed in society is arithmetic, whereas mathematics at the secondary level aims towards advanced academic education. Hirabayashi (1991) characterizes the difference between the two by stating that arithmetic is practical and active, while mathematics is liberal and academic. Fujisawa (1985) states that mathematics is based on theory, unlike arithmetic. Fujisawa states that it is difficult for children to study both arithmetic and mathematics in theory. Therefore, children should focus on studying the theories of mathematics from the continuity of arithmetic and mathematics. Okazaki & Iwasaki (1987) describe the differences of the conditions of the specific operation and visual confirmation between the second level and the third level by the Levels of Geometric Thought (Table1) of Van Hiele (1958). Although the second level takes advantage of the specific operation, the third level considers logical discussion. Levels of Geometric Thought it is not only to know the graphic guidance, it also shows general aspects of the thinking in the studying of mathematics.

Table 1. Levels of Geometric Thought

	Level 0	Level 1	Level 2	Level 3	Level 4
Target of studying	Shapes	Classes of shapes	Properties of shapes	Deductive systems of properties	Analysis of deductive systems
Method of studying	Classes of shapes	Properties of shapes	Deductive systems of properties	Analysis of deductive systems	

Such a situation can be seen from the national curriculum of Japan. The purpose of the Elementary School Arithmetic Commentary at National Curriculum of Japan (Ministry of Education, 2012a) mainly describes activities that work, experiential activities, and activities that use a specific thing. However, those are not only mere activities. Those are activities to consider development and application, activities to represent, and activities to explain. Those activities may have not used a specific thing. For example, those include activities that consider to make a thread while to clear the reason. On the other hand, the important activities of Junior High School Mathematics at National Curriculum of Japan (Ministry of Education, 2012b) describe activities that are found and the development of the property of numbers and figures based on the studied mathematics, activities that use the mathematics on daily life basis in the society, activities that consider to make a thread while to clear the reason using a mathematical representation. These activities are required to consider the event in mathematical.

So far, the author has seen the National Curriculum of Japan a similar trend, which is seen in the United States of Standard (NCTM, 2000) and the UK of National Curriculum (DfEE & QCA, 2004). Although the study of arithmetic and mathematics continues to evolve, there are differences such as those shown in Table2. In Japan, in recent years, continuity in the studying of mathematics has been emphasized. For example, studying of arithmetic is seen in the characteristic of mathematics. It can be seen that arithmetic and mathematics are not separate entities, but parts of a continuing process of learning.

Table 2. The Differences between the Characteristics of the Arithmetic and Mathematics

Arithmetic	Mathematics
Target a specific thing	Target the abstract thing
Specific, direct and work manipulation	Abstract and logical considerations
Practicality and activity	Liberal and academic

So, in order to answer the purpose of this paper, the author considers the proposal of the guidance in the studying of arithmetic at the center of the following three points, which the author discovered as points to remember;

- (1) Arithmetic is in the early stages of studying of arithmetic and mathematics.

- (2) Arithmetic is one of the longest studying of arithmetic and mathematics.
- (3) Studying of arithmetic has characteristics.

Mathematical Representation in the Study of Arithmetic

Nakahara (1995) organized the mathematical representation (Figure 1). Representational System in Mathematical Education position E1.Realistic Representation, E2.Manipulative Representation, I.Illustrative Representation, S1.Linguistic Representation, in this order of S2.Symbolic Representation, from difficult to understandable representation from the easy-to-understand representation, to abstract representation from the specific representation. Furthermore, mathematical representation is acquired earlier than the "I" towards the "E", is acquired earlier than the "S" towards the "I". Each style of representation is a goodness along with its function.

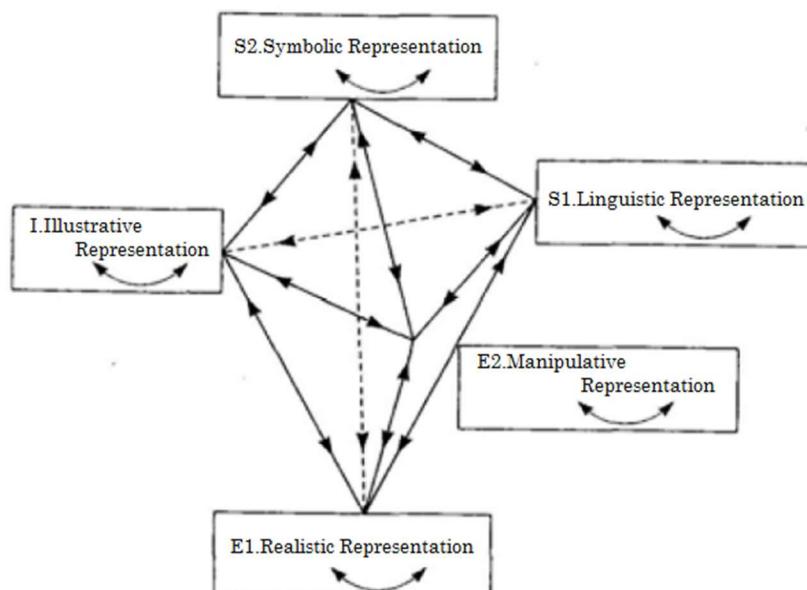


Figure 1. Representational System in Mathematical Education (Nakahara, 1995, p.202)

Arithmetic sets to target specific things; its activities is made by the specific operation, and practicality and living are priority. For this reason, arithmetic uses primarily E1. Realistic Representation, E2.Manipulative Representation, I.Illustrative Representation, and S1.Linguistic Representation. Students study mainly by using S2. Symbolic Representation in mathematics. But children study mainly by using both S1. Linguistic Representation and S2.Symbolic Representation in arithmetic. Finally, the last agreement of studying is mainly by using both S1. Linguistic Representations and S2.Symbolic Representations.

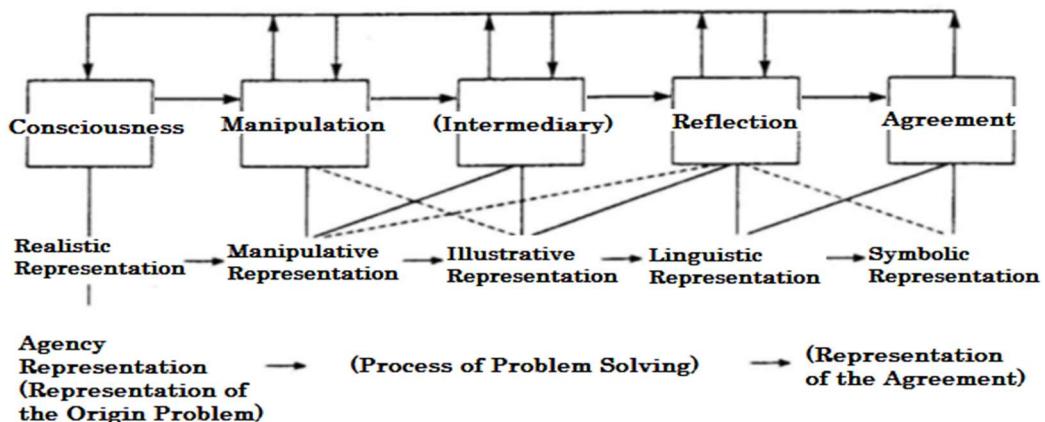


Figure 2. Relationship of Representation Style and Teaching Process Model (Nakahara, 1995)

Shimizu (2012) has already considered the topic of this paper in the study of mathematics, it's at the center of the goals of studying the Linguistic Representation along with the Symbolic Representation in the studying of arithmetic. Also, Illustrative Representation is important to the operation, intermediary and reflection at the study process of arithmetic.

The author focuses on Linguistic Representation and Illustrative Representation in the study of arithmetic in this paper. And, in this paper, we focus on “writing” at a high level of abstraction.

Aspects of the Characteristics of Writing, Analysis of the Difficulty of Writing

According to Vygotsky (1987), in writing, the words are devoid of speech aspects, and describe the characteristic; (1) It needs to notify the meaning of the other partner by its structure of the individual word in sentences by making full use of the force of the character, (2) It is a language activity in the unfamiliar situation of a conversation of a child without the other partner for talking.

The first difficulty is motivation. It is needed when one does not feel the desire to start writing or does not understand why this function is necessary. The second difficulty is voluntary optionality and significance. The writer needs to be aware of the process of representing an idea in words.

The author proposes that just as in the study of mathematics, children not only do not write, but cannot write.

In the next chapter, on the studying of arithmetic, the author considers the point of view of overcoming the difficulties for writing proactively based on two types of difficulty in writing. Also, the author considers points to keep in mind in the teaching of arithmetic, based on differences from the teaching of mathematics described at Shimizu (2012).

Considerations in Overcoming the Difficulty of Writing in the studying of Arithmetic:

From the Point of View of Voluntary Optionality and Significance

As Shimizu (2012) described in the study of mathematics, the studying of arithmetic also uses a variety of representation, and use until Linguistic Representation and Symbolic Representation. So, by

alleviating rules and abstractions of representation, children can go past the awareness of grammar, and insufficient illustration can even convey the meaning to the other party. Children can overcome the situation of being unable to effectively write.

In studying arithmetic, especially in Illustrative Representation and Linguistic Representation, specificity and directness are important. For that reason, the alleviation of rules and abstractions increases the value of Illustrative Representation and is more important than in mathematics study. Also, assuming the study of mathematics occurs after the study of arithmetic, and if children study writing enough, they will improve the effectiveness of their writing. The use of writing in accordance with the situation is important.

Importantly from the above, following points overcome the voluntary optionality and significance in the studying of arithmetic;

- (1) Teachers positively encourage children to write their own ideas, so they can gradually focus on the use of writing.
- (2) First study is not to begin from strict rule representation such as Linguistic Representation, teachers should encourage instead the use of Illustrative Representation that alleviate rules and abstractions in the arithmetic.

These points should be guidance better than studying of mathematics by characteristic of arithmetic.

Considerations for Overcoming the Difficulty of Writing in the Study of Arithmetic:

From the Point of View of Motivation

First, the author considers the problem from the point of view of intrinsic motivation and extrinsic motivation. According to Abe (1995), mathematical representation is the description of the inevitable result that has been backed up by emotion aspects. Isoda (1994) describes that children can grow a sense of efficacy by feeling repeated pleasant experiences based on the studying of McLeod (Figure3). In addition, they improve the hate for arithmetic, and they cultivate good emotions and beliefs. For this reason, Shimizu (2012) described that children need to have continuous positive experiences for the transformation of representation in the studying of mathematics. It's important for the teacher to repeat such experiences while keeping the first experience of the children in mind. And, Shimizu describes the necessity of fun writing to studying itself for children as one of those pleasant experiences.

In the studying of arithmetic, it's important to relate inner surface of children, and do more direct pleasant experiences, for proactive studying from practicality and specificity. One of them is fun and should be emphasized.

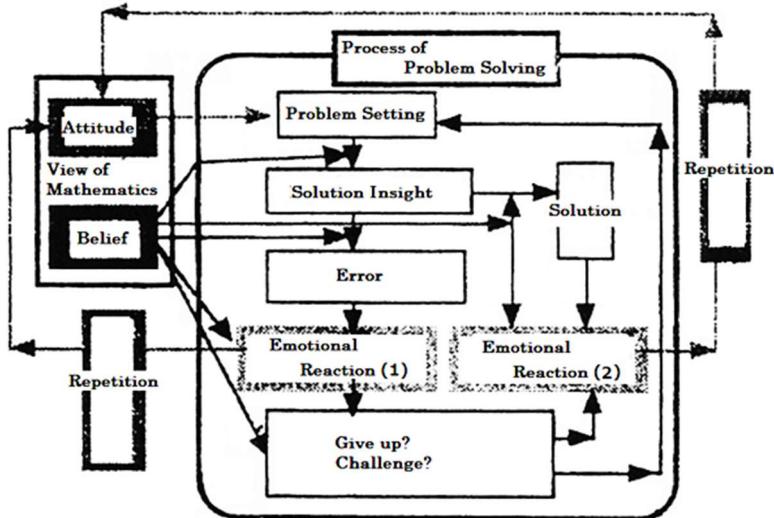


Figure 3. Model for Emotion (Isoda, 1994)

When we consider Isoda's model (Figure 3) again, it is not much of emotion which is a preferable one from the beginning. The emotion gradually becomes preferable. Hayami (1995) describes that it is important to proceed on their own but it may be also a means of having intrinsic motivation. Studying of arithmetic is the initial stage of the studying of long arithmetic and mathematics and the emotion of children is the initial situation. In addition, the belief is difficult to improve and it takes a long time. If changing the emotion and belief as pleasant arithmetic, children must change their emotion more carefully during the long period of studying. Also in the studying of arithmetic, the author emphasizes that the teacher must encourage writing autonomously and mathematically since its purpose doesn't matter to feel the fun of writing. Particularly from children's emotion, early stages of studying, emphasis on practical and specific studying, and continuity to mathematics from arithmetic, studying of arithmetic can be encouraged with very polite guidance and feeling the joy of writing with the contents rather than studying of mathematics.

Next, the author considers from the point of the habitual factors. The author stresses the orientation of writing that children prepare for pleasant situations when thinking of habits. If you think about the long-term continuous studying of arithmetic and mathematics, it is important for orientation of writing that children prepare for pleasant situation and ongoing motivation. In particular, because Japanese children mostly dislike mathematics by international standards, this point of view should be emphasized in the same way as studying of mathematics as studying of arithmetic.

Saeki (1995) describes that children can feel the surprise, excitement and fun when they understand the studying. Surprise and excitement are kinds of emotion that have easy transformation among the affective and they are important in the studying of arithmetic. Saeki (1995) describes that children are changing the perspective by intentions like motivation. Children see themselves as trying to do something, and children can move the settings such as the status of problem. And the intention will change to a person who understands by changing the point of view about things. In order to make

out the intention, teachers should give a feeling of cooperative thinking. This is when one partner changes the studying situation of the other, and the latter follows by doing the reverse.

Children write with a purpose in order to understand. With them, it is important to write the ways, procedures, and reasons until the meaning is completely understood. Then, the teacher has better guidance on how children write for that. The author emphasizes that the orientation of writing has been studied and that children are going to widen their knowledge from within the culture that has been embedded in the studying group by creating situations that can be part amongst the studying population. In particular, a teacher should carefully guide in the studying of arithmetic because it is important in creating an environment of studying the future studying of mathematics as its initial stage.

Considerations for More Proactive Writing in the Studying of Arithmetic

(1) Intentional Setting of Situation for the Studying of Writing

Children acquire knowledge affected by the knowledge of culture in the community of studying. However, for children it can also be difficult. Since studying of arithmetic is the initial stage of studying of arithmetic and mathematics, it is important for children to have teacher's assistance. It is considered to provide appropriate situations of studying. For example, their situation is how to write representation, how to read representation, how to use representation and put the worth of writing. Teachers and children study arithmetic together as a community. It is important for both the teacher and children to feel the fun of arithmetic, wonder of arithmetic, and goodness of writing. And, children use a variety of mathematical representation, also used in writing. In this way, it is important that we feel the worth of fun writing, and the goodness of writing through various aids and own experiences.

This point of view of studying proactive arithmetic is to proactively write arithmetic by feeling its real goodness. Because writing is part of arithmetic studying. It is one of the measures considered to improve Japanese children's interests in arithmetic. In addition, studying arithmetic is the early stage of a long-term arithmetic and mathematics study. The setting of the studying place is a more direct and specific studying technique. Children feel the necessity of writing by providing situations that add a worth of writing for them to write proactively.

(2) Changed Gradually Over Time in Order to Seek the Next Fun and Goodness about Arithmetic

The author motivates the studying of arithmetic by feeling the goodness of mathematics. The author thinks that studying of arithmetic can be more proactive writing because the studying of arithmetic involves writing of arithmetic. Then, the goodness of arithmetic is comprised with fun.

Shimizu (2012) again has discussed the model of Isoda (Figure 3). In that discussion, children's attitudes and beliefs are like the initial value. Moreover, children experience pleasant and good feelings through problem-solving. Thus, children change gradually into having good attitudes and beliefs by obtaining a good emotional reaction.

Yet, the fun is also one of its goodness of a pleasant experience. Emotions are those relatively easier to change than beliefs. Thus, the pleasure of writing is relatively easy to change into the goodness



of writing. Although already mentioned, because the arithmetic studies more specific, practical content as compared to the mathematics, the arithmetic can feel the goodness of arithmetic differently from the goodness of the mathematics. In addition, because the studying of arithmetic treat specific contents, which are about direct contents and based on day-to-day life, arithmetic reaches children's interests. So the studying of arithmetic is easily felt, specifically its goodness over the whole period of the studying of mathematics. If the author thinks in a wider perspective, the studying of arithmetic and mathematics continues long-term if seen from the children's point of view, and the arithmetic as its initial stage, this view should be noted. Thus, it is important for the teacher to use a step-by-step process to keep the fun and goodness of writing in the studying of arithmetic. In other words, the teachers should not exert force. When children are in a situation to want more fun and goodness in a natural way, they should take note to provide a new representation to children. If teachers teach a forced feeling of goodness and use a higher degree representation, there is a possibility to create an arithmetic hater from experiencing the difficulty of representation.

As already mentioned, the author describes that it is important for teachers and children to study together. And, both of them should be involved in the acquisition of knowledge of the studying of arithmetic. Children are to know such representation form the teacher trying to show and use variety of mathematical representations. Then, when children try using a representation, the beginning should be to control writing with some activities, but when the children gradually feels the effect of a representation, writing should then control the activities. In that case, teachers also need a guidance on how to write in a natural way. These activities add to its worth of writing of children. When children are in a situation to have a higher degree of fun and goodness, it's the teacher's duty to provide a new representation for children to use the higher form. Then, the author thinks that they become proactive to write. However, studying of arithmetic is different from the studying of mathematics towards the use of the children's autonomous representation. Teachers should promote various representations, for children to write their own ideas and use Illustrative Representation. The teacher should encourage the use of higher degree representation in a natural context.

Final Remark

The purpose of this paper is to consider the points of view to overcome the difficulties of writing not only the studying of mathematics but also the studying of arithmetic.

First, the author considers the characteristics of arithmetic studying. It was shown by the comparison of the studying of mathematics. Studying of arithmetic is specific and direct. It emphasizes the practicality and activities. However, the studying of arithmetic doesn't eliminate the abstract and academic. Also, studying of arithmetic needs to consider the points of guidance because it is in the early stage of the long studying of arithmetic and mathematics. The author describes two difficulties of writing. And, the author proposed to alleviate the rules and abstractions for its improvement and overcoming the voluntary optionality and significance of each difficulty. The author also proposed that it is good to

feel the fun of writing to be able to understand the purpose of studying and thus improve the motivation. This is the same as the studying of arithmetic as mentioned in the studying of mathematics by Shimizu (2012).

The author proposed the following points of view as guidance for proactive writing in the studying of arithmetic;

- (1) Children should be encouraged to write their own ideas, freely use various kinds of representation, including Illustrative Representation, with the teacher gradually making use of specificity and directness as a priority.
- (2) Teachers should prepare contents which are interesting and understandable, thereby encouraging children's motivation to study autonomously by feeling the enjoyment of writing. The teacher should be aware of the need to carefully progress from the emotional aspect during the earliest stages of studying.
- (3) Both teachers and children should be personally and positively engaged with the objectives of studying. The teacher teaches the way of writing, and children write autonomously.
- (4) In arithmetic activities, the teacher should provide the scene to feel the value of writing, and both of them should see the quality of the value and importance of writing.
- (5) By the children seeking of the higher goodness of arithmetic and higher enjoyment of studying, children should study with the use of new representation from time to time in as natural way as possible.

These points of guidance in the studying of arithmetic should be what teachers need to have more emphasis on in the proposed guidance (1) - (5) rather than studying of mathematics because studying of arithmetic is specific, with its practicality and activities which are the characteristics of arithmetic study. It is also important for the teacher to make good use of the goodness and fun in characteristic of arithmetic. In particular, it is important to emphasize the Illustrative Representation in the studying of arithmetic. Therefore, studying of arithmetic begins with the use of proactive mathematical representation with the strong rules for children to have enough preparation for the reflection of the Illustrative Representation. In addition, studying of arithmetic is the initial stage in the long-term studying of arithmetic and mathematics when seen from children and with those points of view noted by teachers. In this way, guidance on arithmetic study should note at various points as compared to the guidance of mathematics studying (Shimizu, 2012).

Appendix

This paper is intended to clarify the new points should be noted the guidance in arithmetic by considering again from the standpoint of arithmetic through the comparison with the guidance in the studying of mathematics on the basis of the Shimizu (2012).

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